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ESTIMASI PARAMETER MODEL SMITH PADA *MAX-STABLE PROCESS SPATIAL EXTREME VALUE* (Studi Kasus : Pemodelan Curah Hujan Ekstrem di Kabupaten Ngawi)

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PARAMETER ESTIMATION OF SMITH MODEL MAX-STABLE PROCESS SPATIAL EXTREME VALUE (Case Study : Extreme Rainfall Modeling in Ngawi Regency)

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PADA *MAX-STABLE PROCESS SPATIAL EXTREME VALUE*

(Studi Kasus : Pemodelan Curah Hujan Ekstrem di Kabupaten Ngawi)

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ESTIMASI PARAMETER MODEL SMITH PADA *MAX-STABLE PROCESS SPATIAL EXTREME VALUE* (Studi Kasus : Pemodelan Curah Hujan Ekstrem di Kabupaten Ngawi)

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ABSTRAK

Curah hujan ekstrem yang tidak terduga dapat menyebabkan timbulnya bencana banjir. Prediksi curah hujan ekstrem perlu dilakukan agar upaya penanggulangan bencana banjir dapat tepat pada sasaran. Salah satu metode yang dapat memprediksi curah hujan ekstrem yaitu *Spatial Extreme Value* (SEV) dengan pendekatan *Max-Stable Process* (MSP). Salah satu hal yang penting dalam SEV adalah perhitungan *return level* (nilai ekstrem yang diprediksi). Perhitungan *return level* bergantung pada estimasi parameter dalam metode tersebut. Penelitian ini membahas tentang estimasi parameter dari *Spatial Extreme Value Max-Stable Process* khususnya model Smith. Estimasi parameter dilakukan menggunakan metode estimasi *Maximum Composite Likelihood Estimation* (MCLE) dan *Maximum Pairwise Likelihood Estimation* (MPLE). Hasil estimasi menggunakan metode ini tidak *closed form,* sehingga estimasi harus dilanjutkan menggunakan metode iterasi numerik. Metode iterasi yang digunakan pada penelitian ini adalah *Broyden-Fletcher Goldfarb-Shanno* (BFGS) *Quasi Newton,* yang lebih cepat mencapai konvergensi dibandingkan metode lain. Hasil dari estimasi parameter diterapkan pada data curah hujan Kabupaten Ngawi yang merupakan kabupaten dengan produksi pertanian padi terbesar di Provinsi Jawa Timur (provinsi dengan lahan pertanian padi terluas di Indonesia). Berdasarkan hasil analisis data diperoleh model *trend surface* $\hat{\mu}(s) = 2{,}794 + 0{,}242\ v(s)$; $\hat{\sigma}(s) = 1{,}8196 + 0{,}1106\ v(s)$; $\hat{\xi}(s) = 1{,}012$ dengan ukuran kebaikan model *Takeuchi Information Criterion* (TIC) sebesar 26237,62. Nilai *Root Mean Square Error* (RMSE) berdasarkan 20 data testing sebesar 32,078 dan *Mean Absolute Percentage Error* (MAPE) sebesar 27,165%.

Kata kunci : BFGS *Quasi Newton,* curah hujan ekstrem, *likelihood estimation*, *Max-Stable Process*, model Smith, *return level.*

PARAMETER ESTIMATION OF SMITH MODEL MAX-STABLE PROCESS SPATIAL EXTREME VALUE (Case Study : Extreme Rainfall Modeling in Ngawi Regency)

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ABSTRACT

The unpredictable extreme rainfall can affect flood. Prediction of extreme rainfall is needed to do, so that the efforts to preventing the flood can be effective. One of the methods that can predict the extreme rainfall is the Spatial Extreme Value (SEV) with the Max-Stable Process (MSP) approach. The important purpose of SEV is calculate of return level (the extreme value prediction). The calculation of return level depends on parameter estimation in that method. This research discusses about parameter estimation of the Spatial Extreme Value Max-Stable Process especially Smith model. Parameter estimation was performed using Maximum Composite Likelihood Estimation (MCLE) method dan Maximum Pairwise Likelihood Estimation (MPLE) method. The result of estimation using this method is not closed form, it must be continued by using numerical iteration method. The iteration method used in this research is Broyden-Fletcher Goldfarb-Shanno (BFGS) Quasi Newton, which is faster than other methods to achieve convergence. The result of parameter estimation applied to the rainfall data of Ngawi Regency which is the Regency with the largest rice production in East Java Province (the provice with the largest rice farm in Indonesia). Based on the results of data analysis obtained trend surface model $\hat{\mu}(s)$ = 2,794 + 0,242 $v(s)$; $\hat{\sigma}(s)$ = 1,8196 + 0,1106 $v(s)$; $\hat{\xi}(s)$ = 1,012 with goodness criterion model *Takeuchi Information Criterion* (TIC) 26237,62. *Root Mean Square Error* (RMSE) based on 20 testing data is 32,078 and *Mean Absolute Percentage Error* (MAPE) is 27,165%.

Keywords : BFGS Quasi Newton, Smith model, Max-Stable Process, likelihood estimation, extreme rainfall, return level.

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α = taraf signifikansi

a_n, b_n = suatu konstanta

$\boldsymbol{\beta}$ = vektor parameter model *trend surface* ($\boldsymbol{\beta}_{\mu}$, $\boldsymbol{\beta}_{\sigma}$, $\boldsymbol{\beta}_{\xi}$)

$\mathbf{d}$ = vektor koordinat lokasi [1, u, v]

d = dimensi dari lokasi (*longitude* (u), *latitude* (v))

ϵ = elemen dari

ξ = parameter bentuk (*shape*)

$\boldsymbol{\Sigma}$ = matriks kovarian

$\boldsymbol{\Sigma}^{-1}$ = invers dari matriks kovarian

$F(x)$ = fungsi distribusi kumulatif dari variabel X

$F(z_i, z_j)$ = fungsi distribusi kumulatif bivariat dari variabel Z lokasi ke-i dan variabel Z lokasi ke-j

$F^*(x)$ = fungsi distribusi kumulatif data sampel

$F(x)$ = fungsi distribusi kumulatif teoritis

$\boldsymbol{h}_{j,k}$ = vektor jarak antara lokasi j dan lokasi k

i = indeks untuk observasi

j, k = indeks untuk lokasi

$\| \ \|$ = tanda mutlak untuk vektor

m = banyaknya lokasi

μ = parameter lokasi (*location*)

n = jumlah sampel *training* pada suatu variabel

σ = parameter skala (*scale*)

φ = fungsi densitas peluang normal

Φ = fungsi distribusi kumulatif normal

$\theta(\boldsymbol{h}_{j,k})$ = koefisien ekstremal

∞ = tak hingga

p = peluang terjadinya *return level*

r = jumlah sampel *testing* pada suatu variabel

$\mathbb{R}^d$ = ruang berdimensi d

s = lokasi

u_j = *longitude* dari lokasi ke-j

v_j = *latitude* dari lokasi ke-j

X = variabel curah hujan ekstrem

$x_i(s)$ = nilai obserbasi ke-i dari variabel X pada lokasi s

Y = variabel curah hujan harian

Z = variabel hasil transformasi variabel X ke unit margin Frechet

$z_p(s)$ = *return level* pada lokasi s

$\|s_j, s_k\|$ = norm/jarak antara lokasi j dan lokasi k

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BAB I
PENDAHULUAN

1.1. Latar Belakang

Fenomena perubahan iklim menyebabkan munculnya berbagai bencana alam salah satunya adalah banjir. Bencana banjir mengakibatkan kerugian di berbagai bidang, contohnya bidang pertanian, perikanan, dan transportasi. Sebagai Negara agraris, Indonesia menanggung banyak kerugian di bidang pertanian. Jawa Timur adalah salah satu provinsi yang menanggung kerugian tersebut. Jawa Timur merupakan provinsi yang sebagian besar produksi pertaniannya adalah padi, dengan luas lahan sawah mencapai 1.102.863 hektar. Luas lahan sawah provinsi ini merupakan luas lahan sawah terbesar di Indonesia (BPS, 2014).

Terdapat sejumlah kabupaten yang merupakan penghasil padi terbesar di Jawa Timur. Kabupaten Ngawi merupakan salah satu kabupaten yang memiliki lahan pertanian padi cukup besar. Berdasarkan publikasi dari Badan Pusat Statistik (BPS) Jawa Timur, produksi padi Kabupaten Ngawi adalah yang terbesar setelah Jember dan Lamongan. Besar produksi mencapai 770.125 ton dengan luas lahan panen 120.929 ha. Kabupaten Ngawi juga termasuk wilayah yang sering dilanda banjir. Sebanyak 49 desa/kelurahan di Kabupaten Ngawi terkena banjir dan 6 desa/kelurahan dilanda banjir bandang selama tahun 2011 sampai dengan 2013 (BPS, 2014). Curah hujan ekstrem yang tidak terduga adalah penyebab timbulnya bencana ini.

Berdasarkan alasan tersebut prediksi curah hujan ekstrem perlu dilakukan, dengan harapan penanggulangan bencana banjir dapat tepat pada sasaran dan kerugian atas gagalnya pertanian dapat diminimalkan. Salah satu metode untuk memprediksi besarnya curah hujan ekstrem adalah *Spatial Extreme Value* (SEV). metode SEV menangani kasus kejadian ekstrem (seperti curah hujan ekstrem) dengan mempertimbangkan dependensi antara kejadian pada suatu wilayah dengan wilayah lain, seperti pada penelitian Davison, Padoan, dan Ribatet (2012). Nilai dari kejadian-kejadian ekstrem ini diperhitungkan dalam sebuah distribusi *Generalized Extreme Value* (GEV), yaitu distribusi yang memperhatikan bentuk ekor dari distribusi data. Bentuk ekor distribusi mengindikasikan seberapa banyak kejadian ekstrem yang terjadi (Coles, 2001).

SEV dapat didekati dengan *Max-Stable Process* (MSP) (Buishand, De Haan, dan Zhou, 2008). MSP merupakan perluasan dari distribusi *extreme value* multivariat ke dimensi tak hingga (Ribatet, 2009). MSP menggunakan metode *Block Maxima* (BM) dalam pemilihan nilai-nilai ekstrem dari keseluruhan data kasus. Data secara keseluruhan dibagi ke dalam blok-blok interval periode tertentu. Nilai tertinggi pada masing-masing blok merupakan sampel kejadian ekstrem (Gilli dan Kellezi, 2006).

Tujuan utama dari SEV adalah diperolehnya *return level* (nilai prediksi kejadian ekstrem), dalam kasus ini yaitu nilai prediksi curah hujan ekstrem. *Return level* dapat diperoleh apabila sejumlah estimator parameter dari *Cumulative Distribution Function* (CDF) salah satu model dari SEV diketahui. Metode estimasi parameter yang banyak diusulkan pada penelitian sebelumnya untuk memperoleh estimator tersebut adalah *Maximum Composite Likelihood Estimation* (MCLE) dan *Maximum Pairwise Likelihood Estimation* (MPLE) oleh Padoan, Ribatet, dan Sission (2010), Bienvenue dan Robert (2014), serta Blanchet dan Davison (2011). Kedua metode ini mengestimasi parameter dari fungsi dengan variabel dimensi tinggi menggunakan fungsi dengan variabel dimensi rendah, sehingga dalam pengerjaan lebih mudah dibandingkan dengan metode estimasi lainnya. Model-model dalam MSP memiliki bentuk fungsi dengan 2 variabel, sehingga kedua metode estimasi tersebut sangat sesuai dengan kasus SEV yang melibatkan banyak variabel lokasi.

Telah dilakukan pengkajian tentang estimasi parameter *Max-Stable Process* pada penelitian terdahulu, yaitu Ramadhani (2016) tentang model Smith dan *Brown Resnick*, Malika (2015) tentang model *Schlather*, dan Anindita (2015) tentang model Smith. Ketiga penelitian ini belum dilakukan estimasi parameter secara lengkap. Estimasi yang telah dilakukan berhenti pada penulisan fungsi *likelihood* metode MPLE, sehingga perlu dilakukan penelitian lanjutan yang berfokus pada konstruksi dari estimasi parameter *Max-Stable Process* sampai dengan diperolehnya estimator parameter yang diperlukan. Berdasarkan latar belakang tersebut, pada penelitian ini dilakukan estimasi parameter MSP, yang melibatkan salah satu modelnya yaitu model Smith. Model Smith dipilih dengan pertimbangan bahwa model tersebut merupakan model dari MSP yang pertama dikemukakan (Smith, 1990). Model ini juga memiliki *range* koefisien ekstremal (koefisien yang digunakan untuk mengukur seberapa kuat dependensi data antarlokasi) 1 sampai dengan 2 sehingga hasil pengukuran dependensi spasial antarlokasi lebih mudah diintrepretasikan (Schlather dan Tawn, 2003). Apabila hasil estimasi parameter tidak *closed form*, sehingga harus dilanjutkan menggunakan metode iterasi numerik, dalam penelitian ini menggunakan metode itersasi *Broyden-Fletcher Goldfarb-Shanno* (BFGS) *Quasi Newton*, dengan pertimbangan bahwa iterasi pada metode ini lebih cepat mencapai konvergensi dibandingkan metode lain (Chong dan Zak, 1996).

1.2. Perumusan Masalah

Berdasarkan latar belakang di atas, dalam penelitian ini dirumuskan permasalahan yaitu bagaimana proses estimasi parameter model Smith MSP dan bagaimana model curah hujan ekstrem di Kabupaten Ngawi berdasarkan model Smith MSP.

1.3. Tujuan Penelitian

Tujuan dari dilakukannya penelitian ini adalah :

1. Mendapatkan estimator untuk parameter model Smith MSP
2. Mendapatkan model untuk mengestimasi curah hujan ekstrem di Kabupaten Ngawi berdasarkan model Smith MSP.

1.4. Manfaat Penelitian

Penelitian ini memberikan beberapa manfaat. Manfaat tersebut yaitu :

1. Diperolehnya estimator untuk parameter model Smith MSP.
2. Bagi praktisi-praktisi di bidang pertanian dan kelautan, Badan Meteorologi, Klimatologi dan Geofisika (BMKG), juga Lembaga Penerbangan dan Antariksa Nasional (LAPAN), model prediksi curah hujan ekstrem dapat digunakan untuk mengantisipasi timbulnya kerugian yang disebabkan oleh curah hujan ekstrem yang tidak terprediksi.

1.5. Batasan Masalah

Agar tidak terjadi penyimpangan terhadap tujuan penelitian, dalam penelitian ini penulis memberi batasan permasalahan sebagai berikut :

1. Estimasi parameter dilakukan terhadap model Smith karena model ini adalah model pertama yang diperkenalkan dalam MSP dan memiliki *range* ukuran dependensi yang paling mudah diinterpretasikan.
2. Metode estimasi yang digunakan dalam penelitian ini adalah *Maximum Likelihood Estimation* (MLE), MCLE, dan MPLE. Ketiga metode ini merupakan metode yang sejenis sehingga proses estimasi dapat lebih mudah dipahami.
3. Metode iterasi numerik yang digunakan untuk menangani estimasi yang tidak *closed form* adalah Metode BFGS *Quasi Newton*. Metode ini dipilih karena merupakan metode yang paling cepat mencapai konvergensi berdasarkan formulanya dibandingkan metode iterasi numerik lainnya seperti Nelder-Mead.
4. Data yang digunakan adalah data dari 9 pos hujan Kabupaten Ngawi. Hal ini dilakukan karena terdapat banyak data yang hilang pada 16 pos hujan lainnya sehingga jumlah sampel yang dibutuhkan tidak terpenuhi.

BAB II

TINJAUAN PUSTAKA

2.1 *Extreme Value Theory*

Extreme Value Theory (EVT) merupakan teori yang mengkaji tentang peluang kejadian-kejadian ekstrem. Berfokus pada perilaku ekor dari suatu distribusi, teori ini telah digunakan lebih dari 50 tahun (Coles, 2001). Perilaku ekor yang turun secara lambat (bentuk ekor yang gemuk) mengindikasikan adanya peluang terjadinya kejadian ekstrem. Semakin gemuk ekor distribusi, semakin besar pula peluang nilai ekstrem muncul.

Teori ini dapat diterapkan pada berbagai kasus seperti yang telah dikaji oleh McNeil (1999) yaitu untuk manajemen risiko, Coles (2001) pada kasus klimatologi dan hidrologi, serta Smith (2004) pada bidang asuransi dan finansial. Kotz dan Nadarajah (2000) menjelaskan bahwa EVT dapat diterapkan juga pada kasus curah hujan, banjir, badai, dan polusi yang sebagian besar data pada kasus tersebut memiliki ekor distribusi yang turun secara lambat atau disebut *heavy tail*.

McNeil (1999) serta Gilli dan Kellezi (2006) menerangkan bahwa, terdapat dua metode penentuan nilai ekstrem yaitu BM dan *Peak Over Threshold* (POT). BM merupakan metode penentuan nilai ekstrem dengan membuat blok-blok periode (misal blok tiga bulanan atau tahunan). Nilai ekstrem merupakan nilai maksimum yang diambil dari masing-masing blok. POT merupakan metode penentuan nilai ekstrem dengan menentukan suatu nilai *threshold,* dan memilih nilai yang melebihi nilai *threshold* tersebut sebagai nilai ekstrem.

Teori *Extreme Value* melibatkan sebuah distribusi untuk menggeneralisasikan data sampel terpilih yang merupakan nilai-nilai ekstrem. Distribusi ini disebut distribusi GEV. Distribusi ini pertama kali diperkenalkan oleh Jenkinson (1955). X~GEV(μ, α, ξ) memiliki bentuk CDF :

$$F(x;\mu,\sigma,\xi)=\begin{cases}\exp\left(-\left[1+\xi\dfrac{(x-\mu)}{\sigma}\right]^{-1/\xi}\right), -\infty < x < \infty, \xi \neq 0\\ \exp\left(-\exp\left[-\dfrac{(x-\mu)}{\sigma}\right]\right), -\infty < x < \infty, \xi = 0\end{cases} \tag{2.1}$$

Probability Density Function (PDF) distribusi GEV adalah :

$$f(x;\mu,\sigma,\xi)=\begin{cases}\frac{1}{\sigma}\left[1+\xi\frac{(x-\mu)}{\sigma}\right]^{-\frac{1}{\xi}-1}\exp\left(-\left[1+\xi\frac{(x-\mu)}{\sigma}\right]^{-\frac{1}{\xi}}\right),-\infty<x<\infty,\xi\neq 0\\ \frac{1}{\sigma}\exp\left(-\frac{x-\mu}{\sigma}\right)\exp\left(-\exp\left[-\frac{(x-\mu)}{\sigma}\right]\right)-\infty<x<\infty,\xi=0\end{cases} \tag{2.2}$$

x = nilai ekstrem

μ = parameter lokasi (*location*)

σ = parameter skala (*scale*), $\sigma > 0$

ξ = parameter bentuk (*shape*)

Parameter ξ menunjukkan perilaku dari *tail* atau ekor GEV. Distribusi GEV mengikuti distribusi Gumbel ketika ξ = 0, mengikuti distribusi Frechet ketika ξ > 0, dan mengikuti distribusi Weibull ketika ξ < 0. Berikut ini adalah CDF ketiga distribusi tersebut :

a. Distribusi Gumbel (ξ = 0)

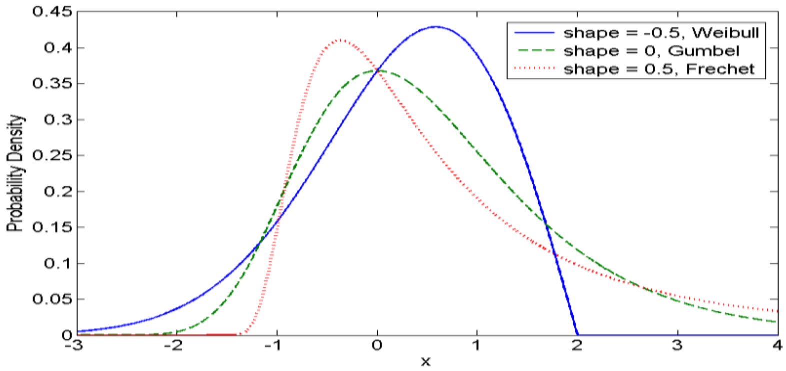
$$F(x;\mu,\sigma)=\exp\left(-\exp\left[-\frac{(x-\mu)}{\sigma}\right]\right),\infty<x<\infty \tag{2.3}$$

b. Distribusi Frechet (ξ > 0)

$$F(x;\mu,\sigma,\xi)=\begin{cases}0 & ,\ x\leq\mu\\ \exp\left[-\left(\frac{x-\mu}{\sigma}\right)^{-\frac{1}{\xi}}\right] & ,x>\mu\end{cases} \tag{2.4}$$

c. Distribusi Weibull (ξ < 0)

$$F(x;\mu,\sigma,\xi)=\begin{cases}\exp\left[-\left\{-\left(\frac{x-\mu}{\sigma}\right)\right\}^{\frac{1}{\xi}}\right] & ,x<\mu\\ 1 & ,x\geq\mu\end{cases} \tag{2.5}$$

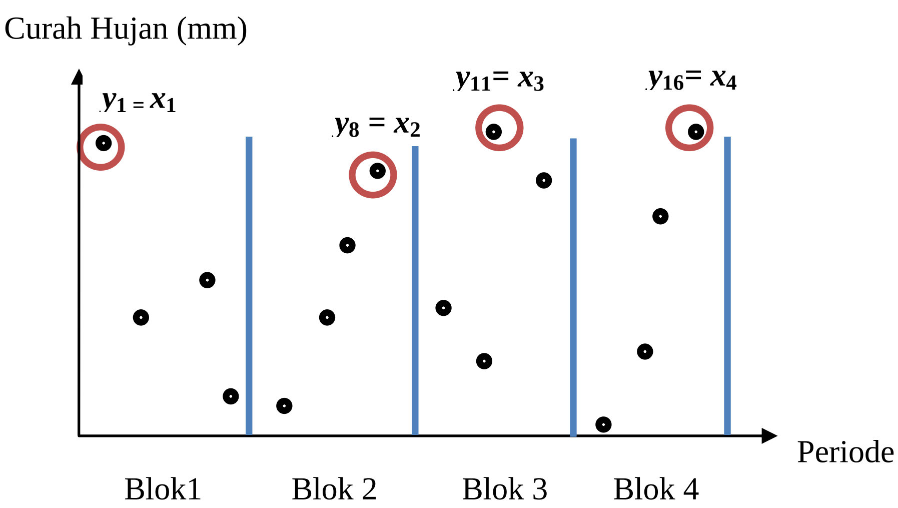


Gambar 2.1 Kurva Distribusi Weibull, Gumbel, dan Frechet

2.2 *Block Maxima*

Block Maxima (BM) merupakan metode penentuan nilai ekstrem yang didasarkan pada pembentukan blok-blok periode. Data observasi dibagi kedalam blok-blok tertentu (misal 3 bulanan atau 6 bulanan). Berdasarkan blok-blok yang terbentuk, dipilih nilai maksimum observasi dari masing-masing blok. Nilai maksimum masing-masing blok yang terpilih disebut sebagai nilai ekstrem blok dan merupakan anggota sampel ekstrem. Metode BM mengaplikasikan teorema Fisher dan Tippet (1928), yang menyatakan bahwa nilai ekstrem yang diambil menggunakan metode BM mengikuti distribusi GEV (Gilli dan Kellezi, 2006).

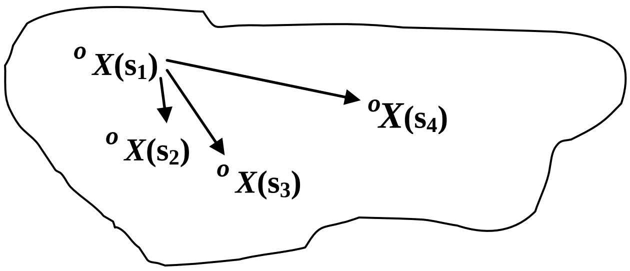
Penerapan metode BM dapat diilustrasikan ke dalam kasus berikut ini. Dimiliki data curah hujan selama satu periode tertentu. Periode ini dibagi ke dalam blok-blok dengan periode yang lebih kecil sesuai pertimbangan dari masing-masing peneliti. Misal satu tahun periode penelitian dibentuk blok tiga bulanan sehingga terbentuk empat blok. Dipilih observasi dengan nilai tertinggi pada masing-masing blok, yaitu observasi y_1, y_8, y_{11}, y_{16}. Observasi-observasi ini yang dijadikan sebagai sampel ekstrem penelitian (X).



Gambar 2.2 Ilustrasi Metode *Block Maxima*

2.3 *Spatial Extreme Value*

Kasus kejadian-kejadian alam khususnya curah hujan, analisis secara univariat tidak cukup. Hal ini dikarenakan kejadian-kejadian alam terjadi bukan hanya pada titik/lokasi tetapi terjadi mencakup area tertentu. Banyak dari data yang bekaitan dengan kejadian alam, merupakan data dari kejadian pada suatu titik dari suatu wilayah, atau data dari suatu wilayah kecil di dalam wilayah yang lebih besar. Berdasarkan data tersebut, dimungkinkan terdapat unsur dependensi antara satu titik dengan titik yang lain dalam satu wilayah kejadian. Analisis terhadap kasus seperti ini teori spasial harus dilibatkan, yaitu dengan mengasumsikan adanya dependensi antarlokasi dalam suatu wilayah kejadian. Metode analisis nilai ekstrem yang melibatkan unsur spasial disebut sebagai metode SEV. Semakin dekat jarak antarlokasi, memungkinkan adanya dependensi yang semakin kuat.



Gambar 2.3 Ilustrasi Efek Spasial

Berdasarkan Gambar 2.3, dapat diilustrasikan bahwa luasan yang tidak beraturan dimisalkan sebuah area atau region yang ingin diteliti dan disimbolkan sebagai $\mathbb{R}^d$. Lingkaran-lingkaran kecil menunjukkan sejumlah lokasi/area yang lebih kecil di dalam $\mathbb{R}^d$. $X(s_1)$ menunjukkan suatu variabel (misalkan variabel curah hujan ekstrem dengan satuan milimeter) pada lokasi(s) ke-1, $X(s_2)$ menunjukkan suatu variabel yang sama pada lokasi(s) ke-2, begitu pula $X(s_3)$ dan $X(s_4)$, sehingga $s \in \mathbb{R}^d$. $d = (\boldsymbol{u}, \boldsymbol{v})$ dengan $\boldsymbol{u}$ adalah *longitude* lokasi dan $\boldsymbol{v}$ adalah *latitude* lokasi. Berdasarkan jarak antara satu lokasi dengan lokasi lain, $X(\mathrm{s}_1)$ dan $X(\mathrm{s}_2)$ memiliki tingkat dependensi yang lebih kuat dibandingkan $X(\mathrm{s}_1)$ dengan $X(\mathrm{s}_3)$. $X(\mathrm{s}_1)$ dan $X(\mathrm{s}_3)$ memiliki tingkat dependensi yang lebih kuat dibandingkan $X(\mathrm{s}_1)$ dengan $X(\mathrm{s}_4)$. Jarak antarlokasi ke-j dan ke-k dapat dihitung menggunakan ukuran jarak $h_{j,k}$ dengan persamaan

$$\boldsymbol{h}_{j,k} = \| \boldsymbol{s}_j - \boldsymbol{s}_k \| \tag{2.6}$$

2.4 *Max-Stable Process*

Max-Stable Proces (MSP) merupakan proses stokastik, perluasan dari distribusi *multivariate extreme value* ke dimensi tak hingga. $\{Z(\boldsymbol{s})\}_{s\epsilon\mathbb{R}^d}$ dikatakan *max-stable* jika suatu konstanta $a_n(\boldsymbol{s}) > 0$ dan $b_n(\boldsymbol{s}) \in$ R, sehingga

$$Z(\boldsymbol{s}) = \lim_{n\to\infty} \frac{\max_{i=1}^{n}\{y_i(\boldsymbol{s})\} - b_n(\boldsymbol{s})}{a_n(\boldsymbol{s})}, n \to \infty, s \in \mathbb{R}^d \tag{2.7}$$

$y_i(\boldsymbol{s})$ berdistribusi random independen identik (Ribatet, 2009).

$Z(\boldsymbol{s})$ dikatakan *max-stable* jika dan hanya jika mengikuti distribusi GEV yang merupakan distribusi untuk data kejadian ekstrem. Bentuk ekor distribusi yang gemuk menunjukkan peluang kejadian ekstrem lebih besar. Distribusi Frechet memiliki bentuk ekor yang paling gemuk dibandingkan distribusi Gumbel dan Weibull. Dengan demikian jika $a_n(\boldsymbol{s}) = n$, $b_n(\boldsymbol{s}) = 0$, $Z(\boldsymbol{s})$ dapat digeneralisasikan ke dalam unit Frechet

$$F(Z) = \exp\left(-\frac{1}{Z}\right), Z > 0$$

Padoan, Ribatet, dan Sisson (2010) menyebutkan bahwa Z merupakan bentuk transformasi dari X, dengan fungsi transformasi

$$Z = \left(1 + \xi\frac{X-\mu}{\sigma}\right)_{+}^{1/\xi} \tag{2.8}$$

dimana $a_+ = \max(0; a)$. Apabila $\{(\xi i, Ui)\}_{i\geq 1}$ adalah proses poisson $(0,\infty] \times \mathbb{R}^d$ dengan intensitas $d\Lambda(\xi, u) = \xi^{-2}(d\xi)\ v(du)$ dan v adalah ukuran σ terbatas pada $\mathbb{R}^d$, MSP $Z(s)$ dapat dijelaskan dengan persamaan

$$Z(s) = \max_{i=1}^{\infty}\{\xi_i\ f_s(U_i)\}_{s\in\mathbb{R}^d} \tag{2.9}$$

2.5 Model Smith

MSP $Z(s)$ dapat ditulis sebagai $Z(s) = \max_{i=1}^{\infty}\{\xi_i\ f_s(U_i)\}_{s\in R^d}$. $\{(\xi i, Ui)\}_{i\geq 1}$ adalah proses poisson $(0,\infty] \times \mathbb{R}^d$ dengan intensitas $d\Lambda(\xi, u) = \xi^{-2}(d\xi)$ $v(du)$. Model Smith dapat dituliskan sebagai $f_s(U_i) = f_s(x_i - s)$, dengan $f_s(x_i - s)$ adalah fungsi densitas normal bivariat. Sehingga $Z(s)$ menjadi (Smith, 1990)

$$Z(s) = \max_{i=1}^{\infty}\{\xi_i\ f_s(x_i - s),\}_{s\in\mathbb{R}^d} \tag{2.10}$$

CDF model Smith adalah

$$F\left(z_j, z_k\right) = exp\left\{-\frac{1}{z_j}\Phi\left(\frac{a\left(h_{j,k}\right)}{2}+\frac{1}{a\left(h_{j,k}\right)}\log\left[\frac{z_k}{z_j}\right]\right)-\frac{1}{z_k}\Phi\left(\frac{a\left(h_{j,k}\right)}{2}+\frac{1}{a\left(h_{j,k}\right)}\log\left[\frac{z_j}{z_k}\right]\right)\right\} \quad (2.11)$$

dengan :

z_j : nilai z lokasi ke-j

z_k : nilai z lokasi ke-k

$\boldsymbol{h_{j,k}}$: vektor jarak antara dua lokasi

Φ : CDF normal standar

$a(\boldsymbol{h_{j,k}})$: $\sqrt{\boldsymbol{h_{j,k}}^T \boldsymbol{\Sigma_{j,k}}^{-1} \boldsymbol{h_{j,k}}}$

$\boldsymbol{\Sigma_{j,k}}$: matriks kovarian dari variabel lokasi ke-j dan ke-k

2.6 Koefisien Ekstremal

Koefisien ekstremal digunakan untuk mengukur tingkat dependensi data ekstrem antara satu wilayah dengan wilayah lain (antara wilayah yang dipasangkan) dalam SEV. Kefisien ekstremal yang disimbolkan sebagai $\Theta(\boldsymbol{h_{j,k}})$, pada model Smith MSP memiliki kisaran nilai $1 \leq \theta\left(\boldsymbol{h_{j,k}}\right) \leq 2$. Nilai $\Theta(\boldsymbol{h_{j,k}})$ semakin mendekati 1 mengindikasikan semakin dependen antara dua variabel tersebut (variabel data ekstrem). $\Theta(\boldsymbol{h_{j,k}})$ semakin mendekati 2 mengindikasikan hubungan antara dua variabel cenderung independen. Fungsi koefisien ekstremal adalah (Schlather dan Tawn, 2003)

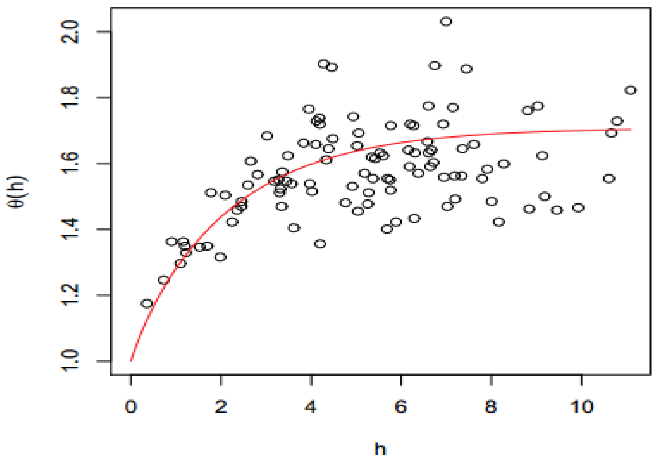
$$\Theta(\boldsymbol{h_{j,k}}) = -z \log P(Z(s_j) \leq z, Z(s_k) \leq z)$$

$$= E\left(max\left[W(s_1), W(s_2)\right]\right) \quad (2.12)$$

$$W(s) = \xi_i\, f_s(x_i - s)$$

Koefisien ekstremal untuk model Smith MSP dihitung menggunakan persamaan berikut

$$\Theta(\boldsymbol{h_{j,k}}) = 2\Phi\left(\frac{\sqrt{\boldsymbol{h_{j,k}}^T \boldsymbol{\Sigma_{j,k}}^{-1} \boldsymbol{h_{j,k}}}}{2}\right) \quad (2.13)$$



Gambar 2.4 Ilustrasi Koefisien Ekstremal (Ribatet, 2011)

2.7 *Takeuchi Information Criterion*

Hasil dari estimasi parameter model Smith MSP menghasilkan nilai $\widehat{\boldsymbol{\beta}}$ yang digunakan untuk membentuk model *trend surface*. Model *trend surface* adalah model linier yang mengkombinasikan variabel koordinat suatu lokasi yaitu berupa variabel *longitude* dan *latitude*, dengan parameter $\boldsymbol{\beta}$. Selanjutnya kombinasi model *trend surface* terbaik dari sembilan kombinasi model digunakan untuk mengestimasi nilai ekstrem pada periode yang akan datang atau yang disebut dengan *return level*. Penentuan model *trend surface* terbaik dilakukan dengan menghitung nilai *Takeuchi Information Criterion* (TIC) yang diusulkan oleh Takeuchi (1976). Kompleksitas model tidak menjamin model tersebut baik. Kombinasi model dengan nilai TIC terkecil yang dipilih sebagai kombinasi model terbaik. Perhitungan nilai TIC adalah sebagai berikut (Padoan, Ribatet, dan Sisson, 2010) :

$$\text{TIC} = -2\, lp(\widehat{\boldsymbol{\beta}}) + 2\, \text{tr}\, [\, \text{H}(\widehat{\boldsymbol{\beta}})^{-1}\, \text{J}(\widehat{\boldsymbol{\beta}})] \qquad (2.14)$$

dengan :

$l_p(\widehat{\boldsymbol{\beta}})$ = fungsi *ln pairwise likelihood* $\sum_{i=1}^{n} \sum_{j=1}^{m-1} \sum_{k=j+1}^{m} ln\left(f\left(z_{ji}, z_{ki}; \widehat{\boldsymbol{\beta}}\right)\right)$

$$H(\widehat{\boldsymbol{\beta}}) = -\frac{\partial^2 l_p(\widehat{\boldsymbol{\beta}})}{\partial \widehat{\boldsymbol{\beta}}\ \partial \widehat{\boldsymbol{\beta}}^{\mathrm{T}}}$$

$$J(\widehat{\boldsymbol{\beta}}) = \sum_{i=1}^{n}\{(\frac{\partial \sum_{j=1}^{m-1} \sum_{k=j+1}^{m} ln\left(f\left(z_{ji}, z_{ki}; \widehat{\boldsymbol{\beta}}\right)\right)}{\widehat{\boldsymbol{\beta}}})\ (\frac{\partial \sum_{j=1}^{m-1} \sum_{k=j+1}^{m} ln\left(f\left(z_{ji}, z_{ki}; \widehat{\boldsymbol{\beta}}\right)\right)}{\widehat{\boldsymbol{\beta}}})^{\mathrm{T}}\}$$

2.8 *Return Level*

Return level adalah nilai maksimum yang dapat dicapai dalam periode tertentu. Nilai ini berarti juga nilai langka/ekstrem yang mungkin terjadi pada periode tertentu yang diprediksi. *Return level* pada lokasi (s) tertentu disimbolkan sebagai $z_p(s)$ yang diestimasi dengan persamaan berikut (Gilli dan Kellezi, 2006) :

$$z_p(s) = \hat{\mu}(s) - \frac{\hat{\sigma}(s)}{\hat{\xi}(s)}\left(1 - \left[-\ln\left(1 - \frac{1}{T}\right)\right]^{-\hat{\xi}(s)}\right) \tag{2.15}$$

dengan :

$\hat{\mu}(s)$ = nilai estimasi parameter lokasi pada lokasi (s) tertentu.

$\hat{\sigma}(s)$ = nilai estimasi parameter skala pada lokasi (s) tertentu.

$\hat{\xi}(s)$ = nilai estimasi parameter bentuk (*shape*) pada lokasi (s) tertentu, disebut juga *tail index.*

T = jumlah blok dalam satu interval periode yang diprediksi.

Peluang tercapainya $z_p(s)$ adalah p = P (Z ≥ z_p) = $\frac{1}{T}$.

2.9 *Maximum Likelihood Estimation*

Fungsi *likelihood* $L(\boldsymbol{\Theta})$ untuk sampel random independen X berukuran n dari PDF $f(x; \boldsymbol{\Theta})$ adalah

$$\begin{aligned} L(\boldsymbol{\Theta}) &= \prod_{i=1}^{n} f(x_i; \boldsymbol{\Theta}) \\ &= f(x_1; \boldsymbol{\Theta})\ f(x_2; \boldsymbol{\Theta}) \ldots\ f(x_n; \boldsymbol{\Theta}) \end{aligned} \tag{2.16}$$

Estimator dari parameter $\boldsymbol{\Theta}$ ($\boldsymbol{\Theta}$ pada penelitian ini adalah parameter $\boldsymbol{\mu}$, $\boldsymbol{\sigma}$, dan $\boldsymbol{\xi}$) diperoleh dari menghitung turunan pertama dari fungsi tersebut dan menyamakannya dengan nol. Hasil dari MLE untuk parameter $\boldsymbol{\Theta}$ merupakan nilai dari $\hat{\boldsymbol{\theta}}$ dimana $L(\hat{\boldsymbol{\theta}}) \geq L(\boldsymbol{\Theta}^*)$. $\boldsymbol{\Theta}^*$ adalah nilai-nilai lain dari $\hat{\boldsymbol{\theta}}$.

Karena L tidak sama dengan nol,

$$\frac{\partial \ln L(\theta)}{\partial \theta} = 0 \tag{2.17}$$

jika dan hanya jika

$$\frac{\partial L(\theta)}{\partial \theta} = 0 \tag{2.18}$$

Dengan demikian dapat diperoleh estimator maksimum dari $\boldsymbol{\Theta}$ secara lebih mudah dengan penurunan *ln* $L(\boldsymbol{\Theta})$ (Kozelka, 1961).

2.10 *Maximum Composite Likelihood Estimation*

Kasus-kasus yang melibatkan banyak variabel, metode estimasi *Full Likelihood* tidak praktis digunakan. Metode *Full Likelihood Estimation* membutuhkan distribusi gabungan yang pada kasus spasial sangat sulit menyusunnya karena banyaknya lokasi yang juga menunjukkan banyaknya variabel. Kasus spasial dengan sebanyak m lokasi membutuhkan distribusi gabungan dari sejumlah m variabel untuk dapat melakukan estimasi menggunakan *Full Likelihood*.

Metode alternatif hasil modifikasi *Full Likelihood* yang dapat digunakan adalah metode MCLE. Metode MCLE dapat menyederhanakan perhitungan secara substansial dan menghasilkan estimator seperti yang diharapkan yaitu bersifat unbias, konsisten, dan normal (Varin, Reid, dan Firth, 2011). Sama seperti MPLE, metode ini pada dasarnya merubah penggunaan PDF berdimensi tinggi ke PDF berdimensi rendah. MPLE melibatkan PDF 2 dimensi (2 variabel), sedangkan MCLE melibatkan PDF 1 dimensi dari variabel yang dianggap independen. Metode *likelihood* ini disebut juga metode *likelihood* independen. Fungsi *composite likelihood* dengan m variabel lokasi dan parameter $\boldsymbol{\beta}$ ($\boldsymbol{\beta}$ pada penelitian ini adalah parameter $\boldsymbol{\beta}_\mu$, $\boldsymbol{\beta}_\sigma$, $\boldsymbol{\beta}_\xi$) dapat dituliskan sebagai (Zhang dan Schneider, 2002) :

$$Lc(\boldsymbol{\beta}) = \prod_{i=1}^{n} L_i(\boldsymbol{\beta})^{w_n} \tag{2.19}$$

dengan w_m adalah pembobot opsional nonnegatif, sehingga apabila w_n = 1, fungsi *composite likelihood* menjadi

$$Lc(\boldsymbol{\beta}) = \prod_{i=1}^{n} L_i(\boldsymbol{\beta}) \tag{2.20}$$

dengan

$L_i(\boldsymbol{\beta}) = \prod_{j=1}^{m} f(x_{ij}; \boldsymbol{\beta})$ (2.21)

Menggunakan prosedur yang sama dengan MLE, agar estimasi lebih mudah fungsi *composite likelihood* diubah terlebih dahulu ke bentuk *ln composite likelihood*. Selanjutnya dilakukan penurunan satu kali terhadap parameter yang diestimasi dan menyamakannya dengan nol.

2.11 *Maximum Pairwise Likelihood Estimation*

MPLE adalah metode estimasi parameter yang menggunakan fungsi densitas *pairwise*/berpasangan dari dua variabel. Seperti halnya metode MLE, estimasi menggunakan metode ini dilakukan dengan menurunkan satu kali fungsi *ln likelihood* terhadap parameter yang diestimasi dan menyamakannya dengan nol. Metode MPLE menggantikan fungsi ($L(\boldsymbol{\beta})$) pada MLE dengan fungsi *pairwise likelihood* $L_p(\boldsymbol{\beta})$.

$L_p(\boldsymbol{\beta}) = \prod_{i=1}^{n} \prod_{j=1}^{m-1} \prod_{k=j+1}^{m} f\left(x_{ji}, x_{ki}; \boldsymbol{\beta}\right)$ (2.22)

$f\left(x_{ji}, x_{ki}; \boldsymbol{\beta}\right)$ merupakan PDF bivariat dengan parameter $\boldsymbol{\beta}$ dan i = 1, 2, … n adalah observasi pada masing-masing variabel.

MSP mentransformasi X ke unit margin Frechet Z dengan fungsi seperti pada persamaan 2.8, sehingga variabel dalam model Smith MSP adalah Z. Estimasi parameter $\boldsymbol{\beta}$ ($\boldsymbol{\beta}_{\mu}$, $\boldsymbol{\beta}_{\sigma}$, $\boldsymbol{\beta}_{\xi}$) hanya dapat diperoleh jika pembentukan fungsi *likelihood* didasarkan pada $f\left(z_{ji}, z_{ki}; \boldsymbol{\beta}\right)$ yang merupakan adalah PDF bivariat model pada MSP (Padoan, Ribatet, dan Sisson, 2010). Ribatet (2009) menuliskan bahwa estimator dari MPLE memenuhi

$\widehat{\boldsymbol{\beta}} \sim \mathrm{N}(\widehat{\boldsymbol{\beta}}, \mathrm{H}(\widehat{\boldsymbol{\beta}})^{-1}\, \mathrm{J}(\widehat{\boldsymbol{\beta}})\, \mathrm{H}(\widehat{\boldsymbol{\beta}})^{-1})$ (2.23)

$l_p(\widehat{\boldsymbol{\beta}})$ = fungsi *ln pairwise likelihood* $\sum_{\mathrm{i}=1}^{\mathrm{n}} \sum_{\mathrm{j}=1}^{\mathrm{m}-1} \sum_{k=j+1}^{m} ln\left(f\left(z_{ji}, z_{ki};\, \widehat{\boldsymbol{\beta}}\right)\right)$

$H(\widehat{\boldsymbol{\beta}}) = -\frac{\partial^2 l_p(\widehat{\boldsymbol{\beta}})}{\partial \widehat{\boldsymbol{\beta}}\, \partial \widehat{\boldsymbol{\beta}}^{\mathrm{T}}}$

$J(\widehat{\boldsymbol{\beta}}) = \sum_{i=1}^{n} \{ \left(\frac{\partial \sum_{\mathrm{j}=1}^{\mathrm{m}-1} \sum_{k=j+1}^{m} ln\left(f\left(z_{ji}, z_{ki}; \widehat{\boldsymbol{\beta}}\right)\right)}{\widehat{\boldsymbol{\beta}}} \right) \left(\frac{\partial \sum_{\mathrm{j}=1}^{\mathrm{m}-1} \sum_{k=j+1}^{m} ln\left(f\left(z_{ji}, z_{ki}; \widehat{\boldsymbol{\beta}}\right)\right)}{\widehat{\boldsymbol{\beta}}} \right)^{\mathrm{T}} \}$

Menggunakan prosedur yang sama dengan MLE, agar estimasi lebih mudah, estimasi parameter fungsi *pairwise likelihood* diubah terlebih dahulu ke bentuk *ln pairwise likelihood.* Selanjutnya dilakukan penurunan satu kali terhadap parameter yang diestimasi dan menyakannya dengan nol.

2.12 *Broyden-Fletcher Goldfarb-Shanno Quasi Newton*

Metode iterasi numerik BFGS *Quasi Newton* merupakan perbaikan dari metode Iterasi *Newton*. Metode ini deperkenalkan oleh Broyden, Fletcher, Goldfarb, dan Shanno pada tahun 1970. Rumus umum metode *Newton Rapson* adalah sebagai berikut (Chong dan Zak, 1996) :

$$\underset{\sim}{\boldsymbol{\theta}}^{(k+1)} = \underset{\sim}{\boldsymbol{\theta}}^{(k)} - \mathbf{H}\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)^{-1} g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right) \tag{2.24}$$

$\underset{\sim}{\boldsymbol{\theta}}^{(k)}$ = nilai awal

$H\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)^{-1}$ = invers dari matriks Hessian

$g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)$ = matriks gradien/matriks yang elemen-elemennya berisi turunan pertama dari fungsi *ln likelihood* terhadap masing-masing parameter.

Dari persamaan rumus umum metode *Newton Rapson* tersebut dilakukan modifikasi menjadi

$$\underset{\sim}{\boldsymbol{\theta}}^{(k+1)} = \underset{\sim}{\boldsymbol{\theta}}^{(k)} - \alpha^{(k)} H\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)^{-1} g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right) \tag{2.25}$$

Metode BFGS *Quasi Newton* mengganti Matriks Hessian $H\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)$ dengan perkiraan yang merupakan matriks definit positif dan memiliki sifat seperti matriks Hessian $H\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)$. Rumus iterasi metode BFGS *Quasi Newton* adalah :

$$\underset{\sim}{\boldsymbol{\theta}}^{(k+1)} = \underset{\sim}{\boldsymbol{\theta}}^{(k)} + \alpha^{(k)} S^{(k)}, \tag{2.26}$$

dimana $\alpha^{(k)}$ merupakan fungsi untuk meminimumkan *error*.

$$\begin{aligned} \alpha^{(k)} &= \arg\min\left[f\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)} + \alpha^{(k)} S^{(k)}\right)\right] \\ &= \frac{S(\theta^{(k)})}{S(\theta^{(k)})^T H(\theta^{(k)}) S(\theta^{(k)})} \end{aligned} \tag{2.27}$$

$$S(\theta^{(k)}) = -H(\theta^{(k)})g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right). \tag{2.28}$$

Kemudian menghitung

$$\Delta\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right) = \alpha^{(k)}S(\theta^{(k)}) \tag{2.29}$$

$$\Delta g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right) = g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k+1)}\right) - g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right) \tag{2.30}$$

sehingga diperoleh persamaan

$$H(\theta^{(k+1)}) = H(\theta^{(k+1)}) + \left(1 + \frac{\Delta g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)^T H(\theta^{(k+1)})\Delta g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)}{\Delta g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)^T \Delta\underset{\sim}{\boldsymbol{\theta}}^{(k)}}\right)\frac{\Delta\underset{\sim}{\boldsymbol{\theta}}^{(k)}\Delta\underset{\sim}{\boldsymbol{\theta}}^{(k)T}}{\Delta\underset{\sim}{\boldsymbol{\theta}}^{(k)T}\Delta g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)} - \frac{H(\theta^{(k+1)})\Delta g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)\Delta\underset{\sim}{\boldsymbol{\theta}}^{(k)T} + \left(H(\theta^{(k+1)})\Delta g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)\Delta\underset{\sim}{\boldsymbol{\theta}}^{(k)T}\right)^T}{\Delta g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)^T \Delta\underset{\sim}{\boldsymbol{\theta}}^{(k)}} \tag{2.31}$$

$H(\theta^{(k)})$ adalah matriks simetris nonsingular. Awal matriks $H(\theta^{(k)})$, yaitu $H(\theta^{(0)})$ dipilih matriks identitas yang diperbarui oleh rumus pembaruan 2.31 pada iterasi berikutnya (Ibrahim, Mamat, dan June. 2014). Iterasi dilakukan hingga $\| \underset{\sim}{\boldsymbol{\theta}}^{(k+1)} - \underset{\sim}{\boldsymbol{\theta}}^{(k)} \| \leq e$ dengan e adalah bilangan yang sangat kecil (Murea, 2005).

2.13 Uji *Anderson Darling*

Uji Anderson Darling adalah suatu uji yang digunakan untuk mengetaui apakah suatu data mengikuti distribusi tertentu (yang dihipotesiskan) atau tidak. Suatu data dapat memenuhi MSP jika dan hanya jika data tersebut berdistribusi GEV. Pengujian distribusi terhadap data ekstrem dapat dilakukan menggunakan uji Anderson Darling dengan prosedur (Engmann dan Cousineau, 2011) :

1. Perumusan hipotesis :

   H_0 : $F(X) = F^*(X)$ (Data mengikuti distribusi teoritis $F^*(X)$)

   H_1 : $F(X) \neq F^*(X)$ (Data tidak mengikuti distribusi teoritis $F^*(X)$)

   Distribusi teoritis dalam penelitian ini GEV.

2. Penentuan Statistik uji

   $$AD = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)\left(ln\left(F^*(x_i)\right) + ln(1 - (F^*(x_{n+1-i})))\right) \tag{2.32}$$

   Keterangan :

   $F(X)$: fungsi distribusi kumulatif data sampel

   $F^*(X)$: fungsi distribusi kumulatif teoritis

   n : ukuran sampel

3. Penentuan kriteria uji

   Kriteria uji menolak H_0 jika nilai AD > nilai kritis yang ditentukan atau *p-value* < α (taraf signifikansi yang telah ditentukan). Nilai kritis ditentukan berdasarkan tabel Anderson Darling.

2.14 *Root Mean Square Error*

Root Mean Square Error (RMSE) merupakan ukuran apakah suatu penaksir merupakan penaksir yang memiliki kinerja yang baik dan layak digunakan. Pengukuran kinerja penaksir dilakukan dengan memperhatikan selisih antara nilai taksiran yang dihasilkan dengan nilai aktual yang diperoleh dari data *testing*. RMSE dirumuskan sebagai berikut (Chai dan Draxler. 2014) :

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{r}(x_i - \hat{x}_i)^2}{r}} \tag{2.33}$$

x_i = nilai aktual (pada penelitian menggunakan data *testing*)

$\hat{x}_i$ = nilai taksiran

r = jumlah sampel untuk data *testing*

Nilai taksiran penelitian ini adalah *return level* z_p yang ditransformasi menjadi x_p.

2.15 *Mean Absolute Percentage Error*

Mean Absolute Percentage Error (MAPE) merupakan ukuran ketepatan relatif yang digunakan untuk mengetahui persentase penyimpangan hasil peramalan. MAPE dihitung dengan persamaan (Sungkawa dan Megasari, 2011) :

$$\text{MAPE} = \frac{1}{r}\sum_{i=1}^{r}|PE_i| \tag{2.34}$$

$$\textit{Percentage Error} \text{ ke-}i\ (PE_i) = \left(\frac{x_i - \hat{x}i}{x_i}\right) \times 100\% \tag{2.35}$$

r = jumlah sampel untuk data *testing*

2.16 Curah Hujan

Curah hujan adalah ketinggian air hujan yang terkumpul dalam penakar datar, tidak menyerap, tidak meresap dan tidak mengalir. Curah satu milimeter artinya dalam luasan satu meter persegi pada tempat yang datar tertampung air hujan setinggi satu milimeter atau tertampung air hujan sebanyak satu liter. Curah hujan ekstrem adalah curah hujan yang memiliki intensitas >100 milimeter per hari. Curah hujan >50 milimeter per hari merupakan curah hujan lebat.

Permulaan musim kemarau, ditetapkan berdasarkan jumlah curah hujan dalam satu dasarian (10 hari) kurang dari 50 milimeter dan diikuti oleh beberapa dasarian berikutnya. Permulaan musim kemarau, bisa terjadi lebih awal (maju), sama, atau lebih lambat (mundur) dari normalnya. Sedangkan permulaan musim hujan ditetapkan berdasarkan jumlah curah hujan dalam satu dasarian (10 hari) sama atau lebih dari 50 milimeter dan diikuti oleh beberapa dasarian berikutnya. Permulaan musim hujan, bisa terjadi lebih awal (maju), sama atau lebih lambat (mundur) dari normalnya (rata-rata dari tahun 1981 - 2010).

Berdasarkan pengelompokan pola distribusi curah hujan rata-rata bulanan di seluruh wilayah Indonesia tahun 1981 - 2010, secara klimatologis wilayah Indonesia terdiri atas 407 pola hujan, 342 pola merupakan Zona Musim (ZOM), sedangkan 65 pola lainnya adalah Non Zona Musim (Non ZOM). ZOM adalah daerah yang pola hujan rata-ratanya mempunyai batas yang jelas secara klimatologis antara periode musim kemarau dan periode musim hujan. Wilayah ZOM tidak selalu sama dengan luas daerah administrasi pemerintahan. Dengan demikian satu kabupaten/kota dapat saja terdiri dari beberapa ZOM dan sebaliknya satu ZOM dapat terdiri dari beberapa kabupaten. Non ZOM adalah daerah-daerah yang tidak mempunyai batas yang jelas secara klimatologis antara periode musim hujan dan musim kemarau (BMKG, 2014). Kabupaten Ngawi masuk pada ZOM 146, 147, dan 152, ditandai dengan garis putih.

Gambar 2.5 Pembagian ZOM Provinsi Jawa Timur (BMKG)

BAB III
METODE PENELITIAN

3.1 Sumber Data

Data yang digunakan dalam penelitian ini merupakan data sekunder yang diperoleh dari Badan Meteorologi, Klimatologi dan Geofisika (BMKG) Karangploso Malang. Data tersebut adalah data curah hujan harian dari 9 pos hujan di Kabupaten Ngawi. Pos-pos hujan tersebut merupakan pos pilihan yang memiliki data curah hujan lengkap selama periode pengamatan dari 25 pos hujan yang ada. Periode pengamatan dalam penelitian ini dimulai dari bulan Maret tahun 1990 sampai dengan bulan November tahun 2015. Terdapat sebanyak 9437 data pengamatan per masing-masing pos.

3.2 Variabel Penelitian

Variabel yang digunakan pada penelitian ini adalah variabel curah hujan dari 9 pos hujan Kabupaten Ngawi. Pos-pos hujan tersebut adalah :

Tabel 3.1 Daftar Pos Hujan Terpilih di Kabupaten Ngawi

No.	Nama Pos Hujan	*Longitude*	*Latitude*
1	Kendal	111,289	-7,560
2	Legundi/Karangjati	111,613	-7,461
3	Gentong/Bekoh	111,301	-7,500
4	Paron	111,396	-7,437
5	Gemarang/Sokongadirejo	111,366	-7,396
6	Kricak	111,344	-7,394
7	Widodaren/Wali Kukun	111,223	-7,385
8	Kedungharjo/Mantingan	111,150	-7,386
9	Guyung	111,369	-7,383

Sumber : Badan Meteorologi Klimatologi dan Geofisika

Gambar 3.1 Peta Lokasi Pos Hujan Kabupaten Ngawi

(Sumber : BMKG Karangploso Malang)

Data yang diperoleh dibagi menjadi dua, yaitu data untuk analisis yang disebut data *training*, data untuk menguji validitas model hasil analisis yang disebut data *testing*. Data *training* menggunakan data periode Maret 1990 sampai dengan November 2010. Data *testing* menggunakan data periode Desember 2010 sampai dengan November 2015. Struktur data pada penelitian ini adalah sebagai berikut :

Tabel 3.2 Struktur Data Penelitian

Tahun	Bulan	Curah Hujan / Observasi	Y_1 (u_1 v_1)	Y_2 (u_2 v_2)	…	Y_9 (u_9 v_9)
1990	3	1	$y_{1,1}$	$y_{2,1}$	…	$y_{9,1}$
1990	3	2	$y_{1,2}$	$y_{2,2}$	…	$y_{9,2}$
1990	3	3	$y_{1,3}$	$y_{2,3}$	…	$y_{9,3}$
1990	3	4	$y_{1,4}$	$y_{2,4}$	…	$y_{9,4}$
1990	3	5	$y_{1,5}$	$y_{2,5}$	…	$y_{9,5}$
1990	3	6	$y_{1,6}$	$y_{2,6}$	…	$y_{9,6}$
⋮	⋮	⋮	⋮	⋮	⋱	⋮
1990	12	304	$y_{1,304}$	$y_{2,304}$	…	$y_{9,304}$
1990	12	305	$y_{1,305}$	$y_{2,305}$	…	$y_{9,305}$
1990	12	306	$y_{1,306}$	$y_{2,306}$	…	$y_{9,306}$
⋮	⋮	⋮	⋮	⋮	⋱	⋮
2015	11	9437	$y_{1,9437}$	$y_{2,9437}$	…	$y_{9,9437}$

Keterangan :

u_j = *longitude* dari lokasi ke-*j*

v_j = *latitude* dari lokasi ke-*j*

3.3 Tahapan Penelitian

Tahapan penelitian yang dilakukan untuk mencapai tujuan penelitan dibagi menjadi dua bagian :

I. Estimasi parameter

a. Estimasi parameter distribusi GEV

Estimasi menggunakan MLE

1. Menyusun fungsi *likelihood* (persamaan 2.16) dari PDF GEV (persamaan 2.2).
2. Membentuk fungsi *ln likelihood*.
3. Melakukan penurunan pertama terhadap fungsi *ln likelihood* dan menyamakannya dengan vektor nol (persamaan 2.17).
4. Apabila hasil estimasi tidak *closed form*, diselesaikan dengan metode BFGS *Quasi Newton*. Proses iterasi didasarkan pada persamaan 2.26, yaitu dengan menghitung $\underset{\sim}{\boldsymbol{\theta}}^{(k+1)}$. Iterasi pertama dimulai dengan k = 0. Iterasi dilakukan dengan mengubah matriks $\underset{\sim}{\boldsymbol{\theta}}^{(k)}$, $\alpha^{(k)}$ (dengan persamaan 2.27), dan $S^{(k)}$ (dengan persamaan 2.28). Iterasi dihentikan apabila hasil yang diperoleh konvergen, atau memenuhi $\| \underset{\sim}{\boldsymbol{\theta}}^{(k+1)} - \underset{\sim}{\boldsymbol{\theta}}^{(k)} \| \leq e$. e merupakan bilangan yang sangat kecil/mendekati nilai nol.

b. Estimasi parameter MSP model Smith

1. Estimasi menggunakan MCLE.
   a) Menyusun PDF bivariat model Smith berdasarkan CDF bivariat model Smith (persamaan 2.11).
   b) Menyusun PDF univariat model Smith dari PDF bivariat model Smith.
   c) Menyusun fungsi *composite likelihood* (persamaan 2.20) dari PDF univariat model Smith.

d) Membentuk fungsi *ln composite likelihood*.

e) Melakukan penurunan pertama terhadap fungsi *ln composite likelihood* dan menyamakannya dengan vektor nol.

f) Apabila hasil dari penurunan pertama tidak *closed form*, dilakukan iterasi numerik menggunakan metode BFGS *Quasi Newton.* Proses iterasi didasarkan pada persamaan 2.25, yaitu dengan menghitung $\underset{\sim}{\boldsymbol{\theta}}^{(k+1)}$. Iterasi pertama dimulai dengan k = 0. Iterasi dilakukan dengan mengubah matriks $\underset{\sim}{\boldsymbol{\theta}}^{(k)}$, $\alpha^{(k)}$ (dengan persamaan 2.27), dan $S^{(k)}$ (dengan persamaan 2.28). Iterasi dihentikan apabila hasil yang diperoleh konvergen, atau memenuhi $\| \underset{\sim}{\boldsymbol{\theta}}^{(k+1)} - \underset{\sim}{\boldsymbol{\theta}}^{(k)} \| \leq e$.

2. Estimasi menggunakan MPLE.

a) Menyusun PDF bivariat model Smith berdasarkan CDF bivariat model Smith (persamaan 2.11).

b) Menyusun fungsi *pairwise likelihood* (persamaan 2.22) dari PDF bivariat model Smith.

c) Menyusun fungsi *ln pairwise likelihood*.

d) Melakukan penurunan pertama terhadap fungsi *ln pairwise likelihood* dan menyamakannya dengan vektor nol.

Apabila hasil dari penurunan pertama tidak *closed form*, dilakukan iterasi numerik menggunakan metode BFGS *Quasi Newton*. Proses iterasi didasarkan pada persamaan 2.25, yaitu dengan menghitung $\underset{\sim}{\boldsymbol{\theta}}^{(k+1)}$. Iterasi pertama dimulai dengan k = 0. Iterasi dilakukan dengan mengubah matriks $\underset{\sim}{\boldsymbol{\theta}}^{(k)}$, $\alpha^{(k)}$ (dengan persamaan 2.27), dan $S^{(k)}$ (dengan persamaan 2.28). Iterasi dihentikan apabila hasil yang diperoleh konvergen, atau memenuhi $\| \underset{\sim}{\boldsymbol{\theta}}^{(k+1)} - \underset{\sim}{\boldsymbol{\theta}}^{(k)} \| \leq e$. e merupakan nilai yang sangat kecil/mendekati nilai nol.

II. Analisis data curah hujan dari 9 pos hujan Kabupaten Ngawi

a. Menghimpun data curah hujan dari 9 pos hujan di Kabupaten Ngawi.

b. Identifikasi adanya data ekstrem.

c. Melakukan analisis deskriptif data.

d. Pengambilan data sampel. Data sampel merupakan nilai-nilai ekstrem dari data *learning*. Penentuan nilai ekstrem berdasarkan metode BM, yaitu dengan membentuk blok-blok dalam interval periode tertentu dari data observasi. Blok-blok dibentuk dengan mengelompokkan data per tiga bulan Desember-Januari-Februari (DJF), Maret-April-Mei (MAM), Juni-Juli-Agustus (JJA), dan September-Oktober-November (SON). Apabila dibagi ke dalam blok 2 bulanan, jumlah nilai ekstrem 0 akan bertambah. Apabila data dibagi ke dalam blok 4 bulanan, jumlah data training hanya ada 55 data setelah dikurangi 20 data testing. Penelitian ini menggunakan data testing sebanyak 20 data dengan pertimbangan yang dijabarkan pada Bab 4. Sampel yang diambil merupakan nilai maksimum dari masing-masing blok.

e. Membagi data menjadi data *training* dan data *testing*. Data *training* merupakan data yang akan dianalisis untuk membentuk model, sedangkan data *testing* digunakan untuk validasi model yang diperoleh. Data *training* dari Maret 1990 – November 2010. Data *testing* dari tahun Desember 2010 – November 2015.

f. Melakukan pengujian distribusi. Menguji kesesuaian distribusi GEV pada data sampel setiap lokasi menggunakan Uji *Anderson Darling*.

g. Melakukan perhitungan nilai $\hat{\mu}(\boldsymbol{s})$, $\hat{\boldsymbol{\sigma}}(\boldsymbol{s})$, dan $\hat{\boldsymbol{\xi}}(\boldsymbol{s})$ secara univariat setiap pos/lokasi.

h. Mentransformasi data sampel ekstrem X ke unit margin Frechet Z menggunakan nilai $\hat{\mu}(\boldsymbol{s})$, $\hat{\boldsymbol{\sigma}}(\boldsymbol{s})$, dan $\hat{\boldsymbol{\xi}}(\boldsymbol{s})$ untuk masing-masing lokasi seperti pada persamaan 2.8.

i. Melakukan pengukuran dependensi antarpos/lokasi dengan menghitung koefisien ekstremal (persamaan 2.13). Pengukuran ini merupakan perhitungan yang mempertimbangkan jarak euclidean antarlokasi berdasarkan koordinat *longitude* dan *latitude*. Koefisien ekstremal semakin mendekati nilai 1 dependensi antarlokasi semakin kuat. Koefisien ekstremal semakin mendekati nilai 2 dependensi antarlokasi semakin lemah.

j. Menyusun dan menentukan kombinasi model *trend surface* terbaik dengan menghitung nilai TIC (persamaan 2.14) untuk setiap kombinasi model *trend surface*.

Tabel 3.3 Kombinasi Model *Trend Surface*

No.	$\mu(s)$	$\sigma(s)$	$\xi(s)$
1	$\beta_{0,\mu} + \beta_{1,\mu} u(s) + \beta_{2,\mu} v(s)$	$\beta_{0,\sigma} + \beta_{1,\sigma} u(s) + \beta_{2,\sigma} v(s)$	$\beta_{0,\xi}$
2	$\beta_{0,\mu} + \beta_{1,\mu} u(s) + \beta_{2,\mu} v(s)$	$\beta_{0,\sigma} + \beta_{1,\sigma} u(s)$	$\beta_{0,\xi}$
3	$\beta_{0,\mu} + \beta_{1,\mu} u(s) + \beta_{2,\mu} v(s)$	$\beta_{0,\sigma} + \beta_{2,\sigma} v(s)$	$\beta_{0,\xi}$
4	$\beta_{0,\mu} + \beta_{1,\mu} u(s)$	$\beta_{0,\sigma} + \beta_{1,\sigma} u(s) + \beta_{2,\sigma} v(s)$	$\beta_{0,\xi}$
5	$\beta_{0,\mu} + \beta_{1,\mu} u(s)$	$\beta_{0,\sigma} + \beta_{1,\sigma} u(s)$	$\beta_{0,\xi}$
6	$\beta_{0,\mu} + \beta_{1,\mu} u(s)$	$\beta_{0,\sigma} + \beta_{2,\sigma} v(s)$	$\beta_{0,\xi}$
7	$\beta_{0,\mu} + \beta_{2,\mu} v(s)$	$\beta_{0,\sigma} + \beta_{1,\sigma} u(s) + \beta_{2,\sigma} v(s)$	$\beta_{0,\xi}$
8	$\beta_{0,\mu} + \beta_{2,\mu} v(s)$	$\beta_{0,\sigma} + \beta_{1,\sigma} u(s)$	$\beta_{0,\xi}$
9	$\beta_{0,\mu} + \beta_{2,\mu} v(s)$	$\beta_{0,\sigma} + \beta_{2,\sigma} v(s)$	$\beta_{0,\xi}$

Sumber : Ramadhani (2016)

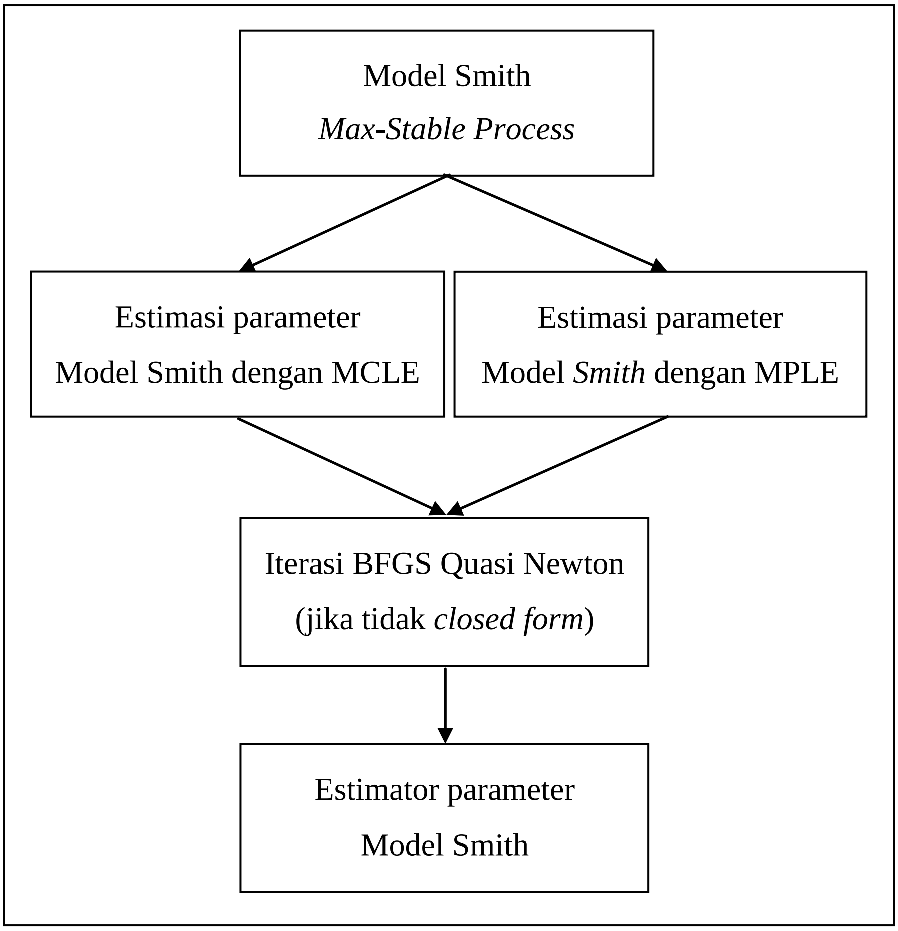
k. Melakukan perhitungan nilai $\hat{\mu}(\boldsymbol{s})$, $\hat{\boldsymbol{\sigma}}(\boldsymbol{s})$, dan $\hat{\boldsymbol{\xi}}(\boldsymbol{s})$ untuk setiap pos berdasarkan model *trend surface* terbaik.

l. Melakukan validasi model dengan menghitung nilai RMSE dan MAPE.

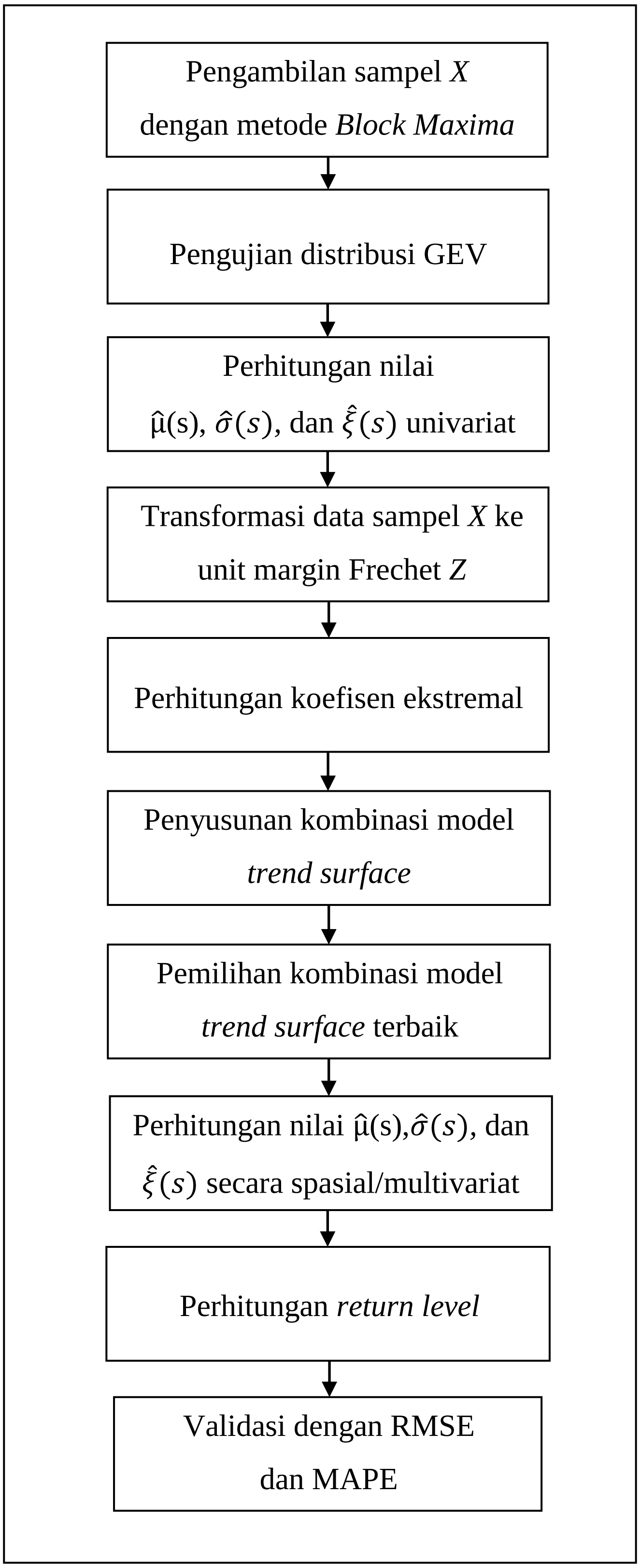
RMSE (persamaan 2.33) dan MAPE (persamaan 2.34) dihitung menggunakan data *testing* dan *return level* (persamaan 2.15) pada periode yang sama dengan data *testing.*

3.4 Diagram Alir Penelitian

Tahapan penelitian untuk mencapai kedua tujuan penelitian dapat digambarkan dalam diagram alir (*flowchart*) berikut ini



Gambar 3.2 Diagram Alir Tujuan Penelitian 1



Gambar 3.3 Diagram Alir Tujuan Penelitian 2

BAB IV
HASIL DAN PEMBAHASAN

4.1 Estimasi Parameter

Perhitungan *return level* pada suatu pos hujan tanpa mempertimbangkan hubungan spasial antarpos hujan melibatkan estimator $\hat{\mu}(s)$, $\hat{\sigma}(s)$, dan $\hat{\xi}(s)$ berdasarkan data curah hujan ekstrem masa lalu pada pos hujan tersebut. Estimator tersebut dapat diestimasi dari fungsi distribusi GEV. Perhitungan *return level* pada suatu lokas/pos hujan (s) tertentu dengan mempertimbangkan hubungan spasial antarpos hujan melibatkan estimator $\hat{\mu}(s)$, $\hat{\sigma}(s)$, dan $\hat{\xi}(s)$. Estimator tersebut dapat dihitung melalui suatu model *trend surface*. Model *trend surface*

$\mu(s) = \beta_{0,\mu} + \beta_{1,\mu}\, u(s) + \beta_{2,\mu}\, v(s)$

$\sigma(s) = \beta_{0,\sigma} + \beta_{1,\sigma}\, u(s) + \beta_{2,\sigma}\, v(s)$

$\xi(s) = \beta_{0,\xi}$

dalam perhitungannya membutuhkan nilai dari parameter $\boldsymbol{\beta}$ (β_μ, β_σ, β_ξ). Parameter ini diperoleh dengan mengestimasi fungsi dari model Smith *Max-Stable Process*.

4.1.1 Estimasi Parameter Distribusi *Generalized Extreme Value*

Estimasi parameter μ, σ, dan ξ dilakukan mengunakan metode estimasi MLE. Proses estimasi parameter μ, σ, dan ξ adalah sebagi berikut :

1. Menyusun fungsi *likelihood* dari PDF distribusi GEV.

   PDF dari GEV adalah

$$f(x;\mu,\sigma,\xi) = \begin{cases} \dfrac{1}{\sigma}\left[1+\xi\dfrac{(x-\mu)}{\sigma}\right]^{-\frac{1}{\xi}-1} \exp\left(-\left[1+\xi\dfrac{(x-\mu)}{\sigma}\right]^{-\frac{1}{\xi}}\right), -\infty < x < \infty, \xi \neq 0 \\ \dfrac{1}{\sigma}\exp\left(-\dfrac{x-\mu}{\sigma}\right)\exp\left(-\exp\left[-\dfrac{(x-\mu)}{\sigma}\right]\right) - \infty < x < \infty, \xi = 0 \end{cases}$$

   MSP dibutuhkan transformasi variabel X ke unit margin frechet Z. Transformasi dilakukan dengan tujuan nilai dari parameter ξ cenderung lebih besar dari nol, yang menyebabkan ekor kanan dari distribusi data semakin gemuk/turun secara lambat. Semakin gemuk ekor distribusi data mengindikasikan semakin besar peluang terjadinya kejadian ekstrem. Ditinjau dari sisi risiko, diperolehnya peluang kejadian ekstrem yang maksimum berarti menanggung risiko maksimum atas terjadinya kejadian ekstrem.

Berdasarkan alasan tersebut, proses estimasi ini menggunakan PDF dari GEV yang berlaku untuk $\xi \neq 0$, yaitu

$$f(x;\mu,\sigma,\xi) = \frac{1}{\sigma}\left[1+\ \xi\ \frac{(x-\mu)}{\sigma}\right]^{-\frac{1}{\xi}-1} \exp\ \left(-\left[1+\ \xi\ \frac{(x-\mu)}{\sigma}\right]^{-1/\xi}\right)$$

Fungsi *likelihood* yang dibentuk dari PDF GEV adalah

$$\begin{aligned}
L(\mu,\sigma,\xi) &= \prod_{i=1}^{n} f(x_i;\mu,\sigma,\xi) \\
&= \prod_{i=1}^{n}\left\{\frac{1}{\sigma}\left[1+\xi\frac{x_i-\mu}{\sigma}\right]^{-\frac{1}{\xi}-1}\exp\left(-\left[1+\xi\frac{x_i-\mu}{\sigma}\right]^{-\frac{1}{\xi}}\right)\right\} \\
&= \left(\frac{1}{\sigma}\right)^{n}\prod_{i=1}^{n}\left(\left[1+\xi\frac{x_i-\mu}{\sigma}\right]^{-\frac{1}{\xi}-1}\right)\exp\left(-\sum_{i=1}^{n}\left[1+\xi\frac{x_i-\mu}{\sigma}\right]^{-\frac{1}{\xi}}\right) \\
&= \left(\frac{1}{\sigma}\right)^{n}\prod_{i=1}^{n}\left(\left[1+\xi\frac{x_i-\mu}{\sigma}\right]^{-\left(\frac{1}{\xi}+1\right)}\right)\exp\left(-\sum_{i=1}^{n}\left[1+\xi\frac{x_i-\mu}{\sigma}\right]^{-\frac{1}{\xi}}\right)
\end{aligned} \tag{4.1}$$

2. Menyusun fungsi *ln likelihood.*

$$\begin{aligned}
\ell(\mu,\sigma,\xi) &= \ln\left[L(\mu,\sigma,\xi)\right] \\
&= \ln\left\{\left(\frac{1}{\sigma}\right)^{n}\prod_{i=1}^{n}\left(\left[1+\xi\frac{x_i-\mu}{\sigma}\right]^{-\left(\frac{1}{\xi}+1\right)}\right)\exp\left(-\sum_{i=1}^{n}\left[1+\xi\frac{x_i-\mu}{\sigma}\right]^{-\frac{1}{\xi}}\right)\right\}
\end{aligned}$$

$$\ell(\mu,\sigma,\xi) = \ln\left\{\sigma^{-n}\prod_{i=1}^{n}\left(\left[1+\xi\frac{x_i-\mu}{\sigma}\right]^{-\left(\frac{1}{\xi}+1\right)}\right)\exp\left(-\sum_{i=1}^{n}\left[1+\xi\frac{x_i-\mu}{\sigma}\right]^{-\frac{1}{\xi}}\right)\right\}$$

$$= -n\ln\sigma + \sum_{i=1}^{n}\ln\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^{-\left(\frac{1}{\xi}+1\right)} - \sum_{i=1}^{n}\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^{-\frac{1}{\xi}} \tag{4.2}$$

$$= -n\ln\sigma - \left(1+\frac{1}{\xi}\right)\sum_{i=1}^{n}\ln\left(1+\xi\frac{x_i-\mu}{\sigma}\right) - \sum_{i=1}^{n}\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^{-\frac{1}{\xi}}$$

3. Penurunan fungsi *ln likelihood* terhadap parameter μ, σ, dan ξ, kemudian menyamakannya dengan nol.

   a. Penurunan terhadap parameter μ

$$\frac{\partial \ell(\mu,\sigma,\xi)}{\partial \mu} = \frac{\partial\left[-n\ln\sigma - \left(1+\frac{1}{\xi}\right)\sum_{i=1}^{n}\ln\left(1+\xi\frac{x_i-\mu}{\sigma}\right) - \sum_{i=1}^{n}\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^{-\frac{1}{\xi}}\right]}{\partial \mu}$$

$$= -\left(1+\frac{1}{\xi}\right)\sum_{i=1}^{n}\left[1+\xi\left(\frac{x_i-\mu}{\sigma}\right)\right]^{-1}\frac{\xi}{\sigma} - \frac{1}{\xi}\sum_{i=1}^{n}\left[1+\xi\left(\frac{x_i-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}-1}\frac{\xi}{\sigma}$$

$$= -\left(\frac{\xi}{\sigma}+\frac{1}{\sigma}\right)\sum_{i=1}^{n}\left[1+\xi\left(\frac{x_i-\mu}{\sigma}\right)\right]^{-1} - \frac{1}{\sigma}\sum_{i=1}^{n}\left[1+\xi\left(\frac{x_i-\mu}{\sigma}\right)\right]^{-\left(\frac{1}{\xi}+1\right)} \tag{4.3}$$

   Turunan pertama terhadap parameter disamakan dengan nol

$$\frac{\partial \ell(\mu,\sigma,\xi)}{\partial \mu} = 0$$

   sehingga diperoleh

$$-\left(\frac{\xi}{\sigma}+\frac{1}{\sigma}\right)\sum_{i=1}^{n}\left[1+\xi\left(\frac{x_i-\mu}{\sigma}\right)\right]^{-1} - \frac{1}{\sigma}\sum_{i=1}^{n}\left[1+\xi\left(\frac{x_i-\mu}{\sigma}\right)\right]^{-\left(\frac{1}{\xi}+1\right)} = 0 \tag{4.4}$$

   b. Penurunan terhadap parameter σ

$$\frac{\partial \ell(\mu,\sigma,\xi)}{\partial \sigma} = \frac{\partial\left[-n\ln\sigma - \left(1+\frac{1}{\xi}\right)\sum_{i=1}^{n}\ln\left(1+\xi\frac{x_i-\mu}{\sigma}\right) - \sum_{i=1}^{n}\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^{-\frac{1}{\xi}}\right]}{\partial \sigma}$$

$$\frac{\partial \ell(\mu,\sigma,\xi)}{\partial \sigma} = -n\left(\frac{1}{\sigma}\right) - \left(1+\frac{1}{\xi}\right)\left(\sum_{i=1}^{n}\left(-\frac{\xi\left(\frac{x_i-\mu}{\sigma^2}\right)}{1+\xi\left(\frac{x_i-\mu}{\sigma}\right)}\right)\right) -$$

$$\left(\sum_{i=1}^{n}\left(-\frac{\left(1+\xi\left(\frac{x_i-\mu}{\sigma}\right)\right)^{1/\xi}\left(\xi\left(\frac{x_i-\mu}{\sigma^2}\right)\right)}{\xi\left(1+\xi\left(\frac{x_i-\mu}{\sigma}\right)\right)}\right)\right) \tag{4.5}$$

Turunan pertama terhadap parameter disamakan dengan nol

$$\frac{\partial \ell(\mu,\sigma,\xi)}{\partial \sigma}=0$$

sehingga diperoleh

$$-n\left(\frac{1}{\sigma}\right)-\left(1+\frac{1}{\xi}\right)\left(\sum_{i=1}^{n}\left(-\frac{\xi\left(\frac{x_i-\mu}{\sigma^2}\right)}{1+\xi\left(\frac{x_i-\mu}{\sigma}\right)}\right)\right)-$$

$$\left(\sum_{i=1}^{n}\left(-\frac{\left(1+\xi\left(\frac{x_i-\mu}{\sigma}\right)\right)^{1/\xi}\left(\xi\left(\frac{x_i-\mu}{\sigma^2}\right)\right)}{\xi\left(1+\xi\left(\frac{x_i-\mu}{\sigma}\right)\right)}\right)\right)=0 \tag{4.6}$$

c. Penurunan terhadap parameter ξ

$$\frac{\partial \ell(\mu,\sigma,\xi)}{\partial \xi}=\frac{\partial\left[-n\ln\sigma-\left(1+\frac{1}{\xi}\right)\sum_{i=1}^{n}\ln\left(1+\xi\frac{x_i-\mu}{\sigma}\right)-\sum_{i=1}^{n}\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^{-\frac{1}{\xi}}\right]}{\partial \xi}$$

$$=\left(\frac{1}{\xi^2}\right)\left[\sum_{i=1}^{n}\ln\left(1+\xi\frac{x_i-\mu}{\sigma}\right)\right]-\left(1+\frac{1}{\xi}\right)\left(\sum_{i=1}^{n}\frac{x_i-\mu}{\sigma\left(1+\xi\frac{x_i-\mu}{\sigma}\right)}\right)-$$

$$\sum_{i=1}^{n}\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^{1/\xi}\left(-\frac{\ln\left(1+\xi\frac{x_i-\mu}{\sigma}\right)}{\xi^2}+\frac{x_i-\mu}{\sigma\xi\left(1+\xi\frac{x_i-\mu}{\sigma}\right)}\right) \tag{4.7}$$

Turunan pertama terhadap parameter disamakan dengan nol

$$\frac{\partial \ell(\mu,\sigma,\xi)}{\partial \xi}=0$$

sehingga diperoleh

$$\left(\frac{1}{\xi^2}\right)\left[\sum_{i=1}^{n}\ln\left(1+\xi\frac{x_i-\mu}{\sigma}\right)\right]-\left(1+\frac{1}{\xi}\right)\left(\sum_{i=1}^{n}\frac{x_i-\mu}{\sigma\left(1+\xi\frac{x_i-\mu}{\sigma}\right)}\right)-$$

$$\sum_{i=1}^{n}\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^{1/\xi}\left(-\frac{\ln\left(1+\xi\frac{x_i-\mu}{\sigma}\right)}{\xi^2}+\frac{x_i-\mu}{\sigma\xi\left(1+\xi\frac{x_i-\mu}{\sigma}\right)}\right)=0 \tag{4.8}$$

Estimasi menggunakan MLE memberikan hasil berupa persamaan yang tidak *closed form*. Persamaan tersebut tidak dapat dirubah ke dalam bentuk yang dapat mengestimasi parameter. Berdasarkan alasan tersebut estimasi parameter harus dilanjutkan menggunakan iterasi numerik. Iterasi numerik yang digunakan dalam penelitian ini adalah BFGS *Quasi Newton*.

Algoritma iterasi BFGS *Quasi Newton* untuk estimasi distribusi GEV adalah :

1. Menentukan nilai awal $\underset{\sim}{\boldsymbol{\theta}}^{(0)}$ yang dapat diisi dengan vektor berukuran px1 dengan seluruh anggotanya adalah nol. p adalah banyaknya parameter yang diestimasi.
2. Menentukan $\alpha^{(k)} = \arg\min\left[f\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)} + \alpha^{(k)} S^{(k)}\right)\right]$
3. Menentukan matriks $H(\theta^{(k+1)})$

$$H(\theta^{(k+1)}) = H(\theta^{(k)}) + \left(1 + \frac{\Delta g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)^T H(\theta^{(k)})\Delta g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)}{\Delta g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)^T \Delta\underset{\sim}{\boldsymbol{\theta}}^{(k)}}\right)\frac{\Delta\underset{\sim}{\boldsymbol{\theta}}^{(k)}\Delta\underset{\sim}{\boldsymbol{\theta}}^{(k)T}}{\Delta\underset{\sim}{\boldsymbol{\theta}}^{(k)T}\Delta g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)} - \frac{H(\theta^{(k)})\Delta g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)\Delta\underset{\sim}{\boldsymbol{\theta}}^{(k)T} + \left(H(\theta^{(k)})\Delta g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)\Delta\underset{\sim}{\boldsymbol{\theta}}^{(k)T}\right)^T}{\Delta g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)^T \Delta\underset{\sim}{\boldsymbol{\theta}}^{(k)}}$$

   $H^{(0)} = I$ (matriks identitas berukuran p x p)

   $$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

   $$\Delta g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right) = g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k+1)}\right) - g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)$$
4. Menentukan $g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)$ yaitu matriks yang elemen-elemennya merupakan turunan pertama dari fungsi *ln likelihood* terhadap masing masing parameter

$$g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right) = \begin{bmatrix} \dfrac{\partial\ell(\mu,\sigma,\xi)}{\partial\mu} \\ \dfrac{\partial\ell(\mu,\sigma,\xi)}{\partial\sigma} \\ \dfrac{\partial\ell(\mu,\sigma,\xi)}{\partial\xi} \end{bmatrix}$$

5. Menentukan $S^{(k)} = -(H^{(k)})g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)$

6. Melakukan iterasi numerik menggunakan persamaan $\underset{\sim}{\boldsymbol{\theta}}^{(k+1)} = \underset{\sim}{\boldsymbol{\theta}}^{(k)} + \alpha^{(k)} S^{(k)}$

7. Menghitung $\Delta\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right) = \underset{\sim}{\boldsymbol{\theta}}^{(k+1)} - \underset{\sim}{\boldsymbol{\theta}}^{(k)}$

8. Kembali ke proses nomor 2 sampai dengan proses nomor 7.

9. Iterasi dimulai dari k = 1 dan dihentikan bila $\| \underset{\sim}{\boldsymbol{\theta}}^{(k+1)} - \underset{\sim}{\boldsymbol{\theta}}^{(k)} \| \leq e$ dengan e adalah bilangan yang sangat kecil.

4.1.2 Estimasi Parameter Model Smith Menggunakan *Maximum Composite Likelihood Estimation*

Berdasarkan penjelasan di awal Bab 4, pada tahapan ini dilakukan estimasi beberapa parameter yang digunakan untuk menyusun suatu model yang disebut model *trend surface.* Parameter yang diestimasi yaitu parameter β_μ, β_σ, dan β_ξ. Berdasarkan rekomendasi dari pustaka yang telah disebutkan pada bagian latar belakang, estimasi parameter yang melibatkan variabel berdimensi tinggi menggunakan fungsi berdimensi rendah dapat diselesaikan menggunakan metode estimasi MCLE, yaitu dengan terlebih dahulu menyusun sebuah fungsi *composite likelihood.* Fungsi *composite likelihood* dengan m variabel dituliskan sebagai

$$Lc(\boldsymbol{\beta}) = \prod_{i=1}^{n} L_i(\boldsymbol{\beta})$$

dengan $L_i(\boldsymbol{\beta}) = \prod_{j=1}^{m} f(x_{ij}; \boldsymbol{\beta})$. Fungsi $L_i(\boldsymbol{\beta}) = \prod_{j=1}^{m} f(x_{ij}; \boldsymbol{\beta})$ merupakan fungsi distribusi gabungan dari m variabel yang independen. Berdasarkan alasan tersebut, metode estimasi ini sering disebut juga metode *likelihood* independen.

Metode dalam penelitian ini mempertimbangkan unsur spasial data, yaitu dengan mengasumsikan adanya dependensi variabel curah hujan pada sebanyak m lokasi. Semakin dekat jarak antarlokasi, diharapkan dependensi semakin kuat. Variabel curah hujan pada sebanyak m lokasi dianggap sebagai m variabel yang dependen. Hal ini tidak sesuai dengan prinsip independen yang terdapat dalam metode MCLE, yaitu melibatkan m variabel yang independen. Berdasarkan pertimbangan tersebut, estimasi parameter model Smith menggunakan metode estimasi MCLE tidak dilanjutkan, sebab estimator yang akan dihasilkan dianggap tidak tepat/tidak sesuai dengan fokus penelitian yang diangkat.

4.1.3 Estimasi Parameter Model Smith Menggunakan *Maximum Pairwise Likelihood Estimation*

Metode estimasi alternatif untuk mengestimasi parameter model Smith adalah metode MPLE. Proses estimasi parameter β_{μ}, β_{σ}, dan β_{ξ} berdasarkan fungsi dari model Smith menggunakan MPLE adalah sebagai berikut :

1. Menyusun PDF model Smith berdasarkan CDF model Smith.

CDF model Smith adalah

$$F\left(z_j, z_k; \beta\right) = exp\left\{-\frac{1}{z_j}\Phi\left(\frac{a\left(h_{j,k}\right)}{2} + \frac{1}{a\left(h_{j,k}\right)}\log\left[\frac{z_k}{z_j}\right]\right) - \frac{1}{z_k}\Phi\left(\frac{a\left(h_{j,k}\right)}{2} + \frac{1}{a\left(h_{j,k}\right)}\log\left[\frac{z_j}{z_k}\right]\right)\right\}$$

Misal

$$w = \frac{a(h_{j,k})}{2} + \frac{1}{a(h_{j,k})}\log\left(\frac{z_k}{z_j}\right)$$

$$v = \frac{a(h_{j,k})}{2} + \frac{1}{a(h_{j,k})}\log\left(\frac{z_j}{z_k}\right)$$

j = 1, 2, …, m-1

k = 2, 3, …, m

CDF model Smith dapat ditulis kembali sebagai

$$F(z_j, z_k; \beta) = \exp\left(-\frac{\Phi(w)}{z_j} - \frac{\Phi(v)}{z_k}\right) \tag{4.9}$$

Penyusunan PDF model Smith dilakukan dengan menurunkan CDF model Smith terhadap variabel yang terlibat di dalamnya yaitu variabel Z_j dan Z_k.

$$\begin{aligned} f(z_j, z_k; \beta) &= \frac{\partial^2}{\partial z_j \partial z_k} F(z_j, z_k; \beta) \\ &= \frac{\partial^2}{\partial z_j \partial z_k}\left\{\exp\left(-\frac{\Phi(w)}{z_j} - \frac{\Phi(v)}{z_k}\right)\right\} \\ &= \exp\left(-\frac{\Phi(w)}{z_j} - \frac{\Phi(v)}{z_k}\right)\left(\frac{\partial}{\partial z_j}\left[-\frac{\Phi(w)}{z_j} - \frac{\Phi(v)}{z_k}\right] \times \right. \\ &\quad \left. \frac{\partial}{\partial z_k}\left[-\frac{\Phi(w)}{z_j} - \frac{\Phi(v)}{z_k}\right] + \frac{\partial^2}{\partial z_j \partial z_k}\left[-\frac{\Phi(w)}{z_j} - \frac{\Phi(v)}{z_k}\right]\right) \end{aligned}$$

$$f(z_j, z_k; \beta) = \exp\left(-\frac{\Phi(w)}{z_j} - \frac{\Phi(v)}{z_k}\right)\left(\left[\frac{\Phi(w)}{z_j^{\ 2}} + \frac{\varphi(w)}{a(h_{j,k})z_j^{\ 2}} - \frac{\varphi(v)}{a(h_{j,k})z_j z_k}\right]\right.$$

$$\left[\frac{\Phi(v)}{z_k^{\ 2}} + \frac{\varphi(v)}{a(h_{j,k})z_k^{\ 2}} - \frac{\varphi(w)}{a(h_{j,k})z_j z_k}\right] + \tag{4.10}$$

$$\left.\left[\frac{v\varphi(w)}{a(h_{j,k})^2 z_j^{\ 2} z_k} - \frac{w\varphi(v)}{a(h_{j,k})^2 z_j z_k^{\ 2}}\right]\right)$$

SEV merupakan metode yang berfokus pada kejadian-kejadian ekstrem. Penelitian ini menganalisis kejadian-kejadian ekstrem yaitu berupa data curah hujan ekstrem yang disimbolkan sebagai *X*, dengan demikian estimasi setiap parameter yang berkaitan dengan analisis didasarkan pada variabel *X*. Pendekatan MSP mentransformasi variabel *X* ke unit margin frechet *Z*. Model Smith MSP mengkombinasikan dua variabel *Z* di dalam fungsi modelnya. Estimasi parameter β_{μ}, β_{σ}, dan β_{ξ} diselesaikan berdasarkan variabel *X*, dengan demikian harus disusun sebuah fungsi baru yaitu fungsi dari variabel *X*. Fungsi dari variabel *X* ini disusun berdasarkan model Smith menggunakan persamaan

2. Menyusun fungsi *pairwise likelihood* dari PDF model Smith. (Lampiran 1)
3. Menyusun fungsi *ln pairwise likelihood*.

Langkah estimasi selanjutnya yaitu menyusun fungsi *ln pairwise likelihood.* (Lampiran 2)

Variabel $\boldsymbol{Z}$ merupakan transformasi dari $\boldsymbol{X}$ dengan fungsi transformasi

$$\boldsymbol{Z} = \left(\mathbf{1} + \boldsymbol{\xi}\,\frac{\boldsymbol{X}-\boldsymbol{\mu}}{\boldsymbol{\sigma}}\right)_{+}^{\mathbf{1}/\boldsymbol{\xi}}$$

Fungsi *ln pairwise likelihood* dapat ditulis kembali dengan menjabarkan variabel *Z* sehingga menjadi (Lampiran 3)

Bentuk dari model *trend surface*

$\mu(s) \;= \beta_{0,\mu} + \beta_{1,\mu}\, u(s) + \beta_{2,\mu}\, v(s)$

$\sigma(s) \;= \beta_{0,\sigma} + \beta_{1,\sigma}\, u(s) + \beta_{2,\sigma}\, v(s)$

$\xi(s) \;= \beta_{0,\xi}$

dapat dituliskan ke dalam bentuk matriks

$\mu(s) = \boldsymbol{d}^{\boldsymbol{T}}\boldsymbol{\beta}_{\boldsymbol{\mu}}$ $\sigma(s) = \boldsymbol{d}^{\boldsymbol{T}}\boldsymbol{\beta}_{\boldsymbol{\sigma}}$ $\xi(s) = \beta_{\xi} = \beta_{0,\xi}$

dengan :

$$\boldsymbol{d}^{\boldsymbol{T}} = \begin{bmatrix} 1 \\ u(s) \\ v(s) \end{bmatrix}$$

$$\boldsymbol{\beta}_{\boldsymbol{\mu}} = \begin{bmatrix} \beta_{0,\mu} \\ \beta_{1,\mu} \\ \beta_{2,\mu} \end{bmatrix}$$

$$\boldsymbol{\beta}_{\boldsymbol{\sigma}} = \begin{bmatrix} \beta_{0,\sigma} \\ \beta_{1,\sigma} \\ \beta_{2,\sigma} \end{bmatrix}$$

$u(s)$ = *longitude* dari suatu lokasi s

$v(s)$ = *latitude* dari suatu lokasi s

Berdasarkan perubahan bentuk parameter μ, ξ, dan σ tersebut, fungsi *ln pairwise likelihood* dapat dijabarkan kembali seperti dalam Lampiran 4.

4. Penurunan fungsi *ln pairwise likelihood* terhadap parameter $\boldsymbol{\beta}_{\mu}$, $\boldsymbol{\beta}_{\sigma}$, dan $\boldsymbol{\beta}_{\xi}$, kemudian menyamakannya dengan nol.
   a. Penurunan terhadap parameter $\boldsymbol{\beta}_{\mu}$ dapat dilihat pada Lampiran 5.
   b. Penurunan terhadap parameter $\boldsymbol{\beta}_{\sigma}$ dapat dilihat pada Lampiran 6.
   c. Penurunan terhadap parameter $\boldsymbol{\beta}_{\xi}$ dapat dilihat pada Lampiran 7.

Estimasi menggunakan MPLE memberikan hasil berupa persamaan yang tidak *closed form*. Persamaan tersebut tidak dapat dirubah ke dalam bentuk yang dapat mengestimasi parameter. Berdasarkan alasan tersebut estimasi parameter harus dilanjutkan menggunakan iterasi numerik. Iterasi numerik yang digunakan dalam penelitian ini adalah BFGS *Quasi Newton*.

Algoritma Iterasi BFGS *Quasi Newton* untuk model Smith adalah sebagai berikut :

1. Menentukan nilai awal $\underset{\sim}{\boldsymbol{\theta}}^{(0)}$ yang dapat diisi dengan vektor berukuran px1 dengan seluruh anggotanya adalah nol. p adalah banyaknya parameter yang diestimasi.

2. Menentukan $\alpha^{(k)} = \arg\min \left[f\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)} + \alpha^{(k)} S^{(k)} \right) \right]$

3. Menentukan matriks $H^{(k+1)}$

$$H^{(k+1)} = H^{(k)} + \left(1 + \frac{\Delta g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)^T H^{(k)} \Delta g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)}{\Delta g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)^T \Delta \underset{\sim}{\boldsymbol{\theta}}^{(k)}} \right) \frac{\Delta \underset{\sim}{\boldsymbol{\theta}}^{(k)} \Delta \underset{\sim}{\boldsymbol{\theta}}^{(k)T}}{\Delta \underset{\sim}{\boldsymbol{\theta}}^{(k)T} \Delta g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)} - \frac{H^{(k)} \Delta g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right) \Delta \underset{\sim}{\boldsymbol{\theta}}^{(k)T} + \left(H^{(k)} \Delta g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right) \Delta \underset{\sim}{\boldsymbol{\theta}}^{(k)T} \right)^T}{\Delta g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)^T \Delta \underset{\sim}{\boldsymbol{\theta}}^{(k)}}$$

dengan

$H^{(0)} = I$ (matriks identitas berukuran p x p)

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

p adalah banyaknya parameter.

$$\Delta g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right) = g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k+1)}\right) - g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)$$

4. Menentukan $g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)$ yaitu matriks yang elemen-elemennya merupakan turunan pertama dari $\underset{\sim}{\boldsymbol{\theta}}^{(k)}$ terhadap masing-masing parameter.

$$g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right) = \begin{bmatrix} \dfrac{\partial \ell(\boldsymbol{\beta})}{\partial \beta_\mu} \\ \dfrac{\partial \ell(\boldsymbol{\beta})}{\partial \beta_\sigma} \\ \dfrac{\partial \ell(\boldsymbol{\beta})}{\partial \beta_\xi} \end{bmatrix}$$

5. Menentukan $S^{(k)} = -(H^{(k)}) g\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right)$

6. Melakukan iterasi numerik menggunakan persamaan

$$\underset{\sim}{\boldsymbol{\theta}}^{(k+1)} = \underset{\sim}{\boldsymbol{\theta}}^{(k)} + \alpha^{(k)} S^{(k)}$$

7. Menghitung $\Delta\left(\underset{\sim}{\boldsymbol{\theta}}^{(k)}\right) = \underset{\sim}{\boldsymbol{\theta}}^{(k+1)} - \underset{\sim}{\boldsymbol{\theta}}^{(k)}$

8. Kembali ke proses nomor 2 sampai dengan proses nomor 7.

9. Iterasi demulai dari k = 1dan dihentikan bila $\| \underset{\sim}{\boldsymbol{\theta}}^{(k+1)} - \underset{\sim}{\boldsymbol{\theta}}^{(k)} \| \le e$ dengan e adalah bilangan yang sangat kecil.

4.2 Analisis Data Curah Hujan

Hasil dari estimasi parameter pada Sub Bab 4.1 diaplikasikan pada data curah hujan harian Kabupaten Ngawi dengan satuan data adalah mm/hari. Pertimbangan penerapan terhadap data telah dijelaskan pada sub bab latar belakang. Kabupaten Ngawi memiliki 25 pos hujan yang tersebar di seluruh wilayah kabupaten. Penelitian ini mengeliminasi 7 pos hujan yang tidak termasuk dalam satu ZOM. Perbedaan ZOM menyebabkan kecenderungan pola hujan berbeda/heterogen, yang dapat menyebabkan analisis spasial tidak tepat. Pos-pos hujan yang tidak termasuk dalam satu Zona Musim ini adalah pos Tretes, Begal, Bekoh, Babadan, Jogorgo, Ngrambe, Kedung Urung-urung. Sembilan pos hujan lainnya dieliminasi dengan pertibangan terlalu banyak data yang irasional pada pos tersebut. Data irasional yang dimaksud seperti halnya data bernilai nol pada lebih dari satu tahun, yang mengakibatkan data tersebut tidak dapat didekati dengan nilai pada tahun tahun sebelumnya. Berdasarkan pertimbangan tersebut, penerapan estimasi pada data curah hujan Kabupaten Ngawi ini hanya melibatkan 9 pos hujan, yaitu Pos Kendal, Legundi, Gentong, Paron, Gemarang, Kricak, Widodaren, Kedungharjo, Guyung. Sekilas data curah hujan 9 pos hujan dapat dilihat pada Lampiran 8.

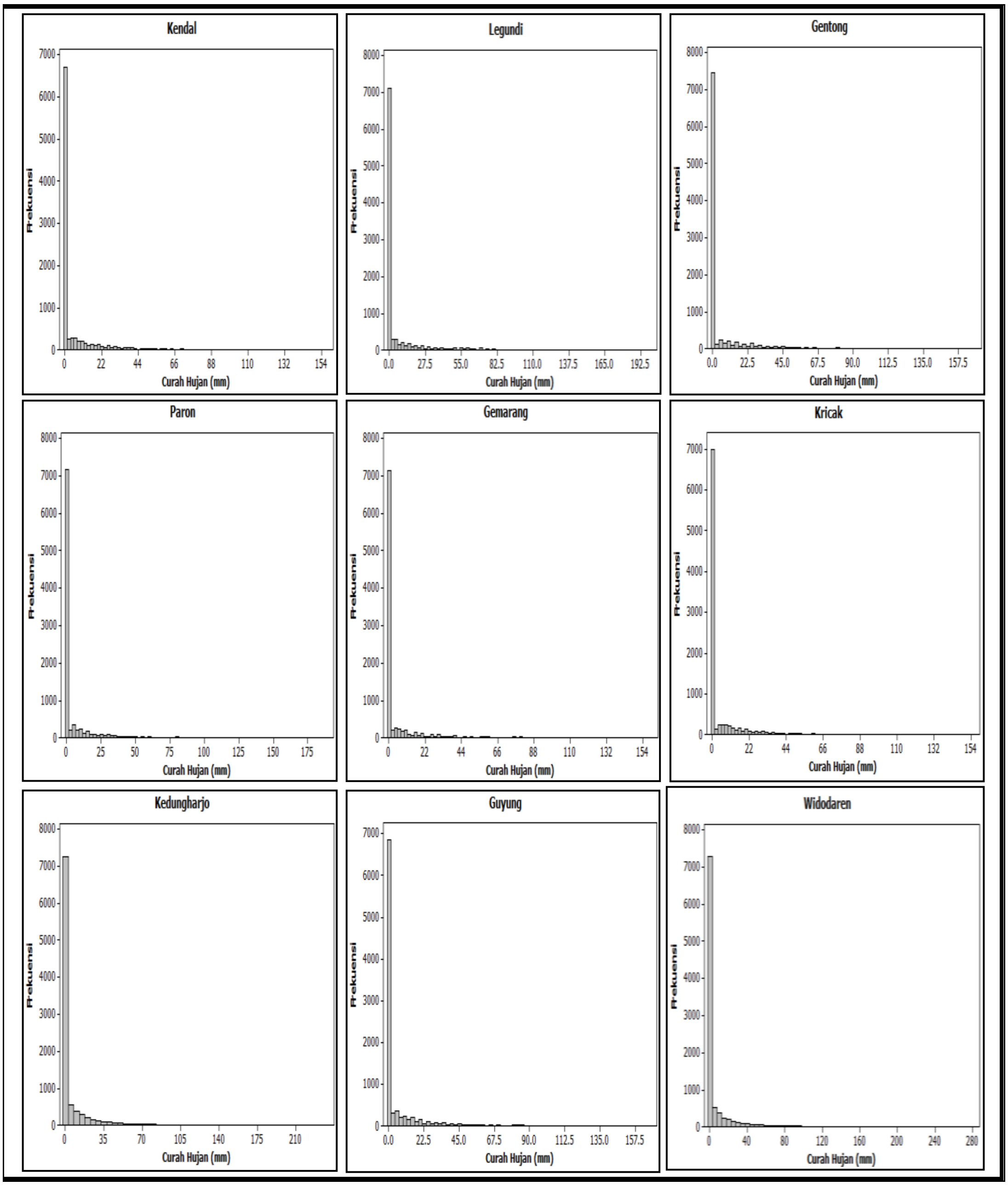
4.2.1 Deskripsi Data

Identifikasi adanya data ekstrem dapat dilakukan dengan menyusun analisis deskritif dari data tersebut. Deskripsi dari data curah hujan Kabupaten Ngawi dari bulan Maret tahun 1990 sampai dengan bulan November tahun 2015 dijelaskan pada Tabel 4.1. Curah hujan minimum adalah nol di seluruh pos hujan, yang artinya tidak ada curah hujan sama sekali dalam satu hari. Curah hujan maksimum sebesar 282 mm dalam satu hari telah terjadi pada pos hujan Widodaren, yang berarti hujan dengan curah terekstrem telah terjadi pada wilayah ini. Pos hujan yang memiliki intensitas curah hujan terendah adalah pos hujan Gemarang dengan mean curah hujan 4,914 mm/hari. Kawasan dengan hujan terlebat adalah kawasan di daerah pos Guyung yang memiliki mean 6,243 mm/hari. Pos ini juga memiliki nilai standar deviasi tertinggi dibandingkan pos hujan lain. Hal ini berarti bahwa curah hujan yang turun pada kawaan pos ini cenderung tidak stabil intensitasnya (keragamannya paling besar). Pos dengan intensitas hujan paling stabil adalah pos Kricak.

Tabel 4.1 Deskripsi Data Curah Hujan Harian 9 Pos Hujan Kabupaten Ngawi

No.	Pos Hujan	Minimum	Maximum	Mean	Standar Deviasi	Skewness	Kurtosis
1	Kendal	0	158	5,902	14,937	3,940	19,865
2	Legundi	0	201	6,164	15,978	3,555	15,567
3	Gentong	0	169	5,114	14,318	4,195	22,993
4	Paron	0	190	5,252	14,435	4,297	23,899
5	Gemarang	0	160	4,914	13,907	4,354	23,440
6	Kricak	0	156	5,039	12,848	4,038	21,785
7	Widodaren	0	282	5,826	15,745	4,050	23,988
8	Kedungharjo	0	240	5,330	14,473	4,380	27,810
9	Guyung	0	168	6,243	16,064	3,833	17,456

Nilai maksimum yang terkecil yaitu 156 mengindikasikan bahwa pada 9 pos hujan di atas telah terjadi hujan yang tergolong ekstrem berdasarkan dengan definisi dari BMKG, yang menyatakan bahwa curah hujan dikategorikan ekstrem apabila mencapai 100mm/hari. Adanya data ekstrem terlihat pula pada rata-rata data masing-masing pos hanya mencapai 4 sampai dengan 6 mm/hari. Alasan diperkuat dengan nilai *skewness* yang diperoleh pada kesembilan pos hujan cukup besar. Nilai *skewness* ini menyatakan bahwa kurva distribusi data cenderung tidak simetri/miring ke salah satu sisi (sisi kanan atau kiri). Dikaitkan dengan analisis secara visual pada histogram data curah hujan masing-masing pos yang terlihat pada gambar 4.1, kurva distribusi data miring ke kanan dan memperlihatkan tingginya frekuensi data menonjol di sekitar nilai nol, sedangkan masih terdapat kejadian dengan curah hujan yang jauh lebih besar dari nol dengan frekuensi yang jauh lebih kecil. Berdasarkan alasan tersebut sembilan pos hujan ini dikategorikan layak menjadi objek penelitian karena merupakan data berekor sehingga dapat dilakukan pengambilan sampel ekstremnya. Sedangkan nilai kurtosis memberikan gambaran seberapa runcing kurva distribusi data. Semakin besar nilai kurtosisnya, semakin runcing kurva, yang mengindikasikan bahwa keragaman data cenderung lebih kecil.



Gambar 4.1 Histogram Data Curah Hujan Harian 9 Pos Hujan

4.2.2 Penentuan Data Sampel

Data sampel pada penelitian ini merupakan nilai-nilai ekstrem dari data curah hujan pada 9 pos hujan di Kabupaten Ngawi. Penentuan sampel dilakukan menggunakan metode BM.

1. Penentuan nilai-nilai ekstrem.

   Penentuan nilai-nilai ekstrem dari data curah hujan pada masing-masing pos hujan menggunakan metode BM. Pemilihan nilai-nilai ekstrem dilakukan dengan membentuk blok-blok tiga bulanan. Blok yang terbentuk adalah blok Desember-Januari-Februari (DJF), Maret-April-Mei (MAM), Juni-Juli-Agustus (JJA), dan September-Oktober-November (SON). Selama 26 tahun periode sampel (1990-2015) terbentuk 103 blok. Diambil satu nilai ekstrem dari setiap blok yang ada. Nilai ekstrem yang diambil merupakan nilai maksimum dari masing-masing blok. Berdasarkan langkah-langkah tersebut terambil 103 data yang merupakan nilai maksimum dari setiap blok tiga bulanan, dari sebanyak 9437 data curah hujan masing-masing pos. 103 data sampel kejadian ekstrem penelitian. Data sampel kejadian ekstrem terlampir pada Lampiran 9 untuk data sampel *training* dan Lampiran 10 untuk data sampel *testing*.

2. Perhitungan parameter $\mu(s), \sigma(s), \xi(s)$ tanpa efek spasial.

   Dilakukan estimasi parameter distribusi GEV terhadap data curah hujan ekstrem masing-masing pos hujan. Parameter tersebut adalah $\mu(s), \sigma(s), \xi(s)$. Parameter ini digunakan untuk mentransformasi data telah memenuhi uji kesesuaian distribusi GEV, ke dalam unit margin frechet. Berikut ini adalah nilai hasil perhitungan estimasi dari ketiga parameter tersebut secara univariat.

Masing-masing parameter yang terdapat distribusi GEV dihitung interval konfidensinya menggunakan pendekatan distribusi normal baku.

$$\hat{\theta} - Z_{\alpha/2}\, se < \theta < \hat{\theta} + Z_{\alpha/2}\, se \qquad (4.11)$$

Perhitungan ini memerlukan nilai *standard error* dari setiap parameter yang diestimasi. Perhitungan nilai *standard error* melibatkan varians parameter yang membutuhkan turunan kedua dari bentuk *ln likelihood* dari fungsi yang digunakan untuk mengestimasi parameter (lihat persamaan 2.23). Turunan kedua dari fungsi *ln likelihood* distribusi GEV terdapat pada Lampiran 18. Berikut ini nilai estimasi parameter $\mu(s), \sigma(s),$ dan $\xi(s)$ menggunakan toleransi *error* α = 5%.

Tabel 4.2 Nilai Estimator $\hat{\mu}$*(s)* Univariat Masing-masing Pos Hujan

No.	Pos Hujan	Koefisien	*Standard Error*	Interval Konfidensi	
				Lower	***Upper***
1	Kendal	61,349	4.699	52,140	70,558
2	Legundi/Karangjati	54,735	4.624	45,674	63,797
3	Gentong/Bekoh	56,721	4.670	47,568	65,873
4	Paron	54,341	4.558	45,406	63,275
5	Gemarang/Sokongadirejo	55,273	4.643	46,173	64,373
6	Kricak	50,246	4.243	41,930	58,561
7	Widodaren/Wali Kukun	57,786	4.190	49,574	65,998
8	Kedungharjo/Mantingan	54,081	4.324	45,607	62,556
9	Guyung	59,719	4.511	50,876	68,561

Parameter μ*(s)* merupakan parameter lokasi yang menyatakan letak titik pemusatan data. Tabel 4.2 menunjukkan nilai estimasi pemusatan data terbesar terdapat pada data ekstrem pos hujan Kendal, yang berarti bahwa intensitas curah hujan ekstrem pada pos hujan ini cenderung lebih besar dari curah hujan ekstrem pos hujan lain. Pemusatan data dengan nilai estimasi terkecil adalah data ekstrem pada pos hujan Kricak. Intensitas curah hujan ekstrem pada pos ini cenderung lebih kecil dari curah hujan ekstrem pos hujan lainnya.

Tabel 4.3 Nilai Estimator $\hat{\sigma}(s)$ Univariat Masing-masing Pos Hujan

No.	Pos Hujan	Koefisien	*Standard Error*	Interval Konfidensi	
				Lower	***Upper***
1	Kendal	39,129	3,341	32,582	45,677
2	Legundi/Karangjati	38,604	3,175	32,380	44,828
3	Gentong/Bekoh	38,801	3,263	32,405	45,197
4	Paron	37,473	3,262	31,080	43,867
5	Gemarang/Sokongadirejo	38,423	3,309	31,938	44,908
6	Kricak	34,874	3,005	28,984	40,764
7	Widodaren/Wali Kukun	35,372	2,805	29,875	40,870
8	Kedungharjo/Mantingan	35,881	3,002	29,997	41,764
9	Guyung	37,984	3,174	31,763	44,204

Parameter $\boldsymbol{\sigma}(s)$ merupakan parameter skala yang menyatakan keragaman data. Pos hujan yang memiliki nilai keragaman data ekstrem terbesar terhadap nilai pemusatan datanya adalah data pos hujan Kendal, yang berarti curah hujan ekstrem pada pos hujan Kendal memiliki *range* persebaran data terbesar, intensitas curah hujan terjadi lebih beragam dibanding dengan pos hujan lain. Pos hujan yang memiliki keragaman data ekstrem terkecil adalah pos hujan Kricak, yang berarti *range* persebaran data ekstrem pada pos hujan ini terkecil, intensitas curah hujan terjadi tidak terlalu beragam dibanding dengan pos hujan lain. Nilai $\hat{\boldsymbol{\sigma}}(s)$ yang diperoleh sesuai dengan nilai $\hat{\mu}(s)$. Data dengan $\hat{\mu}(s)$ terbesar memiliki $\hat{\boldsymbol{\sigma}}(s)$ terbesar pula. Data dengan $\hat{\mu}(s)$ terkecil memiliki $\hat{\boldsymbol{\sigma}}(s)$ terkecil pula.

Tabel 4.4 Nilai Estimator $\hat{\boldsymbol{\xi}}(s)$ Univariat Masing-masing Pos Hujan

No.	**Pos Hujan**	**Koefisien**	***Standard Error***	**Interval Konfidensi**	
				Lower	***Upper***
1	Kendal	-0,336	0,065	-0,464	-0,209
2	Legundi/Karangjati	-0,184	0,057	-0,296	-0,073
3	Gentong/Bekoh	-0,266	0,064	-0,391	-0,140
4	Paron	-0,268	0,073	-0,411	-0,124
5	Gemarang/Sokongadirejo	-0,295	0,069	-0,429	-0,160
6	Kricak	-0,232	0,071	-0,372	-0,092
7	Widodaren/Wali Kukun	-0,078	0,041	-0,158	0,001
8	Kedungharjo/Mantingan	-0,079	0,060	-0,197	0,039
9	Guyung	-0,302	0,054	-0,408	-0,197

Parameter $\boldsymbol{\xi}(s)$ merupakan parameter bentuk yang menyatakan bagaimana perilaku dari ekor kanan distribusi data. Semakin besar nilai parameter $\boldsymbol{\xi}(s)$, ekor kanan cenderung turun secara landai (bentuk ekor gemuk). Semakin gemuk ekor distribusi data maka peluang terjadinya kejadian ekstrem semakin besar. Pos hujan yang memiliki peluang terjadi curah hujan ekstrem terbesar adalah pos hujan Widodaren, yang berarti bahwa pada wilayah ini lebih sering terjadi hujan dengan curah ekstrem dibanding dengan pos hujan lain. Pos hujan yang memiliki peluang terjadinya hujan ekstrem paling kecil terdapat pada pos hujan Kendal.

4.2.3 Pengujian Kesesuaian Distribusi

Data dapat didekati dengan MSP apabila data mengikuti distribusi GEV. Pengujian kesesuaian distribusi dalam penelitian ini dilakukan menggunakan Uji Anderson Darling dengan hipotesis sebagai berikut

H_0 : $F(X) = F^*(X)$ (Data mengikuti distribusi GEV)

H_1 : $F(X) \neq F^*(X)$ (Data tidak mengikuti distribusi GEV)

Berikut ini adalah rekapitulasi hasil pengujian kesesuaian distribusi GEV dari data curah hujan ekstrem Kabupaten Ngawi.

Tabel 4.5 Pengujian Kesesuaian Distribusi

No.	Pos Hujan	Statistik Uji		Kesimpulan
		AD Hitung	*P-Value*	
1	Kendal	0,665	0,946	Data mengikuti distribusi GEV
2	Legundi	0,913	0,994	Data mengikuti distribusi GEV
3	Gentong	0,495	0,851	Data mengikuti distribusi GEV
4	Paron	0,225	0,239	Data mengikuti distribusi GEV
5	Gemarang	0,730	0,968	Data mengikuti distribusi GEV
6	Kricak	0,504	0,865	Data mengikuti distribusi GEV
7	Widodaren	1,642	0,999	Data mengikuti distribusi GEV
8	Kedungharjo	0,691	0,971	Data mengikuti distribusi GEV
9	Guyung	0,535	0,879	Data mengikuti distribusi GEV

Nilai AD Hitung dibandingkan dengan nilai kritis yang ditentukan berdasarkan Tabel Anderson Darling (AD Tabel) pada Lampiran 25. Menggunakan toleransi *error* $\alpha = 5\%$ dan $n = 83$ diperoleh nilai AD Tabel sebesar 2,49849. Nilai AD Hitung dari data curah hujan masing-masing pos lebih kecil dari 2,49849 dan *P-Value* masing-masing pos lebih besar dari α (5%). Berdasarkan perolehan hasil tersebut, data curah hujan ekstrem dari 9 pos hujan di Kabupaten Ngawi mengikuti distribusi GEV.

4.2.4 Pengukuran Dependensi

Analisis terhadap kasus-kasus spasial memperhatikan unsur dependensi data antarwilayah, yaitu dengan mengasumsikan bahwa semakin dekat jarak antarwilayah maka semakin kuat dependensi data antarwilayah tersebut.

Pengukuran dependensi antarpos/lokasi dengan menghitung koefisien ekstremal (persamaan 2.13). Pengukuran ini merupakan perhitungan jarak euclidean antarlokasi berdasarkan koordinat *longitude* dan *latitude*. Nilai koefisien ekstremal semakin mendekati nilai 1 dependensi antarlokasi semakin kuat. Koefisien ekstremal semakin mendekati nilai 2 dependensi antarlokasi semakin lemah. Hasil perhitungan koefisien ekstremal terdapat pada Tabel 4.6.

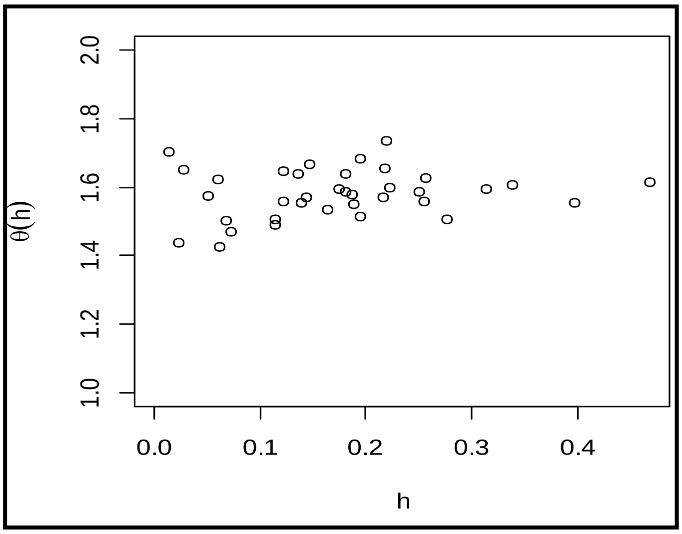
Tabel 4.6 Koefisien Ekstremal Antarlokasi Pos Hujan

Pasangan Lokasi ke-	**Jarak Euclidean Antarlokasi**	**Koefisien Ekstremal**	***Standard Error***	**Interval Konfidensi**	
				Lower	***Upper***
1	0,33879	1,46205	0,05847	1,347	1,577
2	0,06119	1,35327	0,04771	1,260	1,447
3	0,16303	1,40418	0,06509	1,277	1,532
4	0,18118	1,50307	0,07079	1,364	1,642
5	0,17487	1,47940	0,06763	1,347	1,612
6	0,18703	1,44790	0,06753	1,316	1,580
7	0,22270	1,51293	0,07285	1,370	1,656
8	0,19424	1,60888	0,10083	1,411	1,807
9	0,31443	1,45030	0,06021	1,332	1,568
10	0,21832	1,51517	0,07033	1,377	1,653
11	0,25541	1,44126	0,06245	1,319	1,564
12	0,27722	1,42082	0,05355	1,316	1,526
13	0,39734	1,46440	0,06724	1,333	1,596
14	0,46904	1,51496	0,06955	1,379	1,651
15	0,25616	1,50296	0,07441	1,357	1,649
16	0,11399	1,38219	0,05314	1,278	1,486
17	0,12264	1,43525	0,05587	1,326	1,545
18	0,11439	1,40586	0,06451	1,279	1,532
19	0,13896	1,41918	0,05976	1,302	1,536
20	0,18920	1,45697	0,06298	1,334	1,580
21	0,13533	1,52707	0,07902	1,372	1,682
22	0,05080	1,47663	0,06473	1,350	1,603
23	0,06748	1,41754	0,06415	1,292	1,543
24	0,18065	1,42364	0,06558	1,295	1,552
25	0,25123	1,46464	0,06584	1,336	1,594
26	0,06037	1,53153	0,08199	1,371	1,692
27	0,02209	1,37016	0,04980	1,273	1,468

Tabel 4.6 (Lanjutan)

Pasangan Lokasi ke-	Jarak Euclidean Antarlokasi	Koefisien Ekstremal	*Standard Error*	Interval Konfidensi	
				Lower	*Upper*
28	0,14342	1,46607	0,07250	1,324	1,608
29	0,21623	1,44511	0,06537	1,317	1,573
30	0,01334	1,58721	0,09887	1,393	1,781
31	0,12133	1,51902	0,07470	1,373	1,665
32	0,19416	1,41957	0,06274	1,297	1,543
33	0,02731	1,52814	0,08824	1,355	1,701
34	0,07301	1,36450	0,06480	1,237	1,492
35	0,14601	1,54022	0,07921	1,385	1,695
36	0,21902	1,58641	0,08584	1,418	1,755

Gambar 4.2 adalah ilustrasi dari hasil perhitungan koefisien ekstremal 36 pasang lokasi, dengan *h* adalah jarak, dan $\Theta(h)$ adalah koefisien ekstremal. Grafik memperlihatkan bahwa hubungan antarlokasi berdasarkan jarak kurang menonjol, yaitu koefisien ekstremal yang memiliki nilai di kisaran 1,5. Grafik memperlihatkan bahwa semakin jauh jarak semakin kecil keterkaitan datanya.



Gambar 4.2 Grafik Koefisien Ekstremal Curah Hujan Ekstrem 9 Pos Hujan

Hubungan jarak dan koefisien ekstremal pada penelitian ini berbanding positif secara lambat/tidak terlalu signifikan, ini terlihat dari kemiringan persebaran data ke arah kanan atas yang sangat landai.

4.2.5 Pembentukan Model *Trend Suface*

4.2.5.1 Transformasi Data

Berdasarkan tinjauan pustaka pada Bab 2 penelitian ini, analisis data menggunakan metode MSP perlu dilakukan transformasi data terlebih dahulu yaitu dari unit data yang berdistribusi GEV ke unit margin frechet. Hal ini dilakukan agar distribusi data cenderung memiliki ekor kanan dari distribusi yang gemuk/turun secara lambat. Semakin gemuk ekor distribusi semakin besar peluang terjadinya kejadian ekstrem. Peluang maksimum yang diperoleh merupakan wujud dari pemaksimuman risiko atas terjadinya curah hujan ekstrem. Transformasi dengan melibatkan ketiga parameter GEV yang telah dihitung secara univariat menghasilkan data transformasi pada Lampiran 11.

4.2.5.2 Penentuan Kombinasi Model *Trend Surface* Terbaik

Kompleksitas suatu model tidak menjamin bahwa model tersebut semakin baik. Terdapat Sembilan kombinasi model *trend surface* dalam penelitian ini. Estimasi terbaik atas curah hujan ekstrem dapat dilakukan menggunaka kombinasi model terbaik dari sembilan kombinasi model yang ada. Suatu kombinasi model *trend surface* dikatakan terbaik dari kombinasi model *trend surface* lainnya apabila kombinasi model tersebut memiliki nilai TIC terkecil. Hasil perhitungan nilai TIC dari sembilan kombinasi model *trend surface* terdapat pada Tabel 4.4.

Tabel 4.7 Nilai TIC Kombinasi Model *Trend Surface*

Kombinasi ke-	Kombinasi Model	TIC
1	$\hat{\mu}(s) = 14{,}903 - 0{,}112\ u(s) + 0{,}195\ v(s)$ $\hat{\sigma}(s) = 8{,}195 - 0{,}059\ u(s) + 0{,}089\ v(s)$ $\hat{\xi}(s) = 1{,}041$	26329,20
2	$\hat{\mu}(s) = 15{,}208 - 0{,}128\ u(s)$ $\hat{\sigma}(s) = 8{,}195 - 0{,}059\ u(s) + 0{,}089\ v(s)$ $\hat{\xi}(s) = 1{,}041$	26358,00
3	$\hat{\mu}(s) = 14{,}903 - 0{,}112\ u(s) + 0{,}195\ v(s)$ $\hat{\sigma}(s) = 8{,}335 - 0{,}066\ u(s)$ $\hat{\xi}(s) = 1{,}041$	26305,97

Tabel 4.7 (Lanjutan)

Kombinasi ke-	Kombinasi Model	TIC
4	$\hat{\mu}(s)$ = 15,208 – 0,128 $u(s)$ $\hat{\sigma}(s)$ = 8,335 – 0,066 $u(s)$ $\hat{\xi}(s)$ = 1,041	26324,67
5	$\hat{\mu}(s)$ = 2,717 + 0,232 $v(s)$ $\hat{\sigma}(s)$ = 8,188 – 0,055 $u(s)$ + 0,137 $v(s)$ $\hat{\xi}(s)$ = 1,041	26297,68
6	$\hat{\mu}(s)$ = 14,903 – 0,112 $u(s)$ + 0,192 $v(s)$ $\hat{\sigma}(s)$ = 1,795 + 0,111 $v(s)$ $\hat{\xi}(s)$ = 1,041	26339,31
7	$\hat{\mu}(s)$ = 2,794 + 0,242 $v(s)$ $\hat{\sigma}(s)$ = 1,820 + 0,111 $v(s)$ $\hat{\xi}(s)$ = 1,012	26237,62
8	$\hat{\mu}(s)$ = 15,209 – 0,128 $u(s)$ $\hat{\sigma}(s)$ = 1,795 + 0,111 $v(s)$ $\hat{\xi}(s)$ = 1,041	26381,75
9	$\hat{\mu}(s)$ = 2,717 + 0,232 $v(s)$ $\hat{\sigma}(s)$ = 8,335 – 0,066 $u(s)$ $\hat{\xi}(s)$ = 1,041	26270,50

Kombinasi model yang memiliki nilai TIC terkecil adalah kombinasi model keempat dengan nilai TIC 26237,62 yaitu model *trend surface*

$$\hat{\mu}(s) = 2{,}794 + 0{,}242\, v(s)$$
$$\hat{\sigma}(s) = 1{,}820 + 0{,}111\, v(s)$$
$$\hat{\xi}(s) = 1{,}012$$

Masing-masing parameter yang terdapat di dalam model *trend surface* dihitung interval konfidensinya menggunakan pendekatan distribusi normal baku (persamaan 4.11). Perhitungan ini memerlukan nilai *standard error* dari setiap parameter yang diestimasi. Perhitungan *standard error* dari $\hat{\boldsymbol{\beta}}$ melibatkan varians yang membutuhkan turunan kedua dari fungsi *ln likelihood* dari fungsi dari model Smith. Turunan kedua dari fungsi *ln likelihood* model Smith terdapat pada lampiran 19, 20, 21, 22, 23, dan 24. Nilai interval konfidensi dari $\hat{\boldsymbol{\beta}}$ menggunakan toleransi *error* α = 5% adalah sebagai berikut.

Tabel 4.8 Interval Konfidensi Estimator Parameter $\boldsymbol{\beta}$ Model *Trend Surface*

Estimator	Koefisien	*Standard Error*	Interval Konfidensi	
			Lower	*Upper*
$\hat{\boldsymbol{\beta}}_{\mu 0}$	2,794	4,093	-5,227	10,815
$\hat{\boldsymbol{\beta}}_{\mu 1}$	0,242	0,550	-0,836	1,320
$\hat{\boldsymbol{\beta}}_{\sigma 0}$	1,820	6,813	-11,534	15,173
$\hat{\boldsymbol{\beta}}_{\sigma 1}$	0,111	0,917	-1,687	1,908
$\hat{\boldsymbol{\beta}}_{\xi 0}$	1,012	0,048	0,919	1,105

4.2.5.3 Perhitungan Parameter Model dengan Efek Spasial

Perhitungan *return level* memerlukan parameter $\mu(s)$, $\sigma(s)$, dan $\xi(s)$. Berikut ini adalah nilai estimasi dari ketiga parameter tersebut yang dihitung menggunakan model *trend surface* terbaik.

Tabel 4.9 Nilai Estimator $\hat{\mu}(\boldsymbol{s})$, $\hat{\boldsymbol{\sigma}}(\boldsymbol{s})$, dan $\hat{\boldsymbol{\xi}}(\boldsymbol{s})$ secara Multivariat

No.	Pos Hujan	$\hat{\mu}(s)$	$\hat{\sigma}(s)$	$\hat{\xi}(s)$
1	Kendal	0,965	0,983	1,012
2	Legundi/Karangjati	0,988	0,994	1,012
3	Gentong/Bekoh	0,979	0,990	1,012
4	Paron	0,994	0,997	1,012
5	Gemarang/Sokongadirejo	1,004	1,001	1,012
6	Kricak	1,005	1,002	1,012
7	Widodaren/Wali Kukun	1,007	1,003	1,012
8	Kedungharjo/Mantingan	1,007	1,003	1,012
9	Guyung	1,007	1,003	1,012

4.2.6 Validasi Model

Validasi model dengan menghitung nilai RMSE, yang mengukur selisih antara nilai aktual dan estimasi. RMSE dihitung menggunakan data *testing* dan *return level* (persamaan 2.15) pada periode yang sama dengan data *testing.* Perhitungan *return level* menggunakan nilai estimasi parameter Tabel 4.9 untuk periode lima tahun memperoleh hasil yang dapat dilihat pada Tabel 4.10.

Tabel 4.10 Nilai *Return Level* Frechet

Periode	*Return Level*				
	Kendal	**Legundi**	**Gentong**	**Paron**	**Gemarang**
1 tahun	3.421	3,472	3,452	3,484	3,506
2 tahun	7,444	7,540	7,502	7,564	7,603
3 tahun	11,485	11,626	11,571	11,661	11,719
4 tahun	15,541	15,727	15,654	15,773	15,850
5 tahun	19,609	19,840	19,749	19,896	19,992
Periode	***Return Level***				
	Kricak	**Widodaren**	**Kedungharjo**	**Guyung**	
1 tahun	3,507	3,511	3,511	3,512	
2 tahun	7,605	7,614	7,613	7,616	
3 tahun	11,722	11,735	11,733	11,738	
4 tahun	15,853	15,870	15,869	15,874	
5 tahun	19,997	20,018	20,016	20,023	

Tabel 4.11 Nilai *Return Level Generalized Extreme Value*

Periode	*Return Level*				
	Kendal	**Legundi**	**Gentong**	**Paron**	**Gemarang**
1 tahun	100,759	97,679	97,684	94,100	95,566
2 tahun	118,456	119,864	117,268	112,879	113,939
3 tahun	126,491	130,941	126,567	121,787	122,522
4 tahun	131,440	138,163	132,447	127,420	127,896
5 tahun	134,916	143,450	136,658	131,450	131,714
Periode	***Return Level***				
	Kricak	**Widodaren**	**Kedungharjo**	**Guyung**	
1 tahun	88,212	100,091	96,982	99,416	
2 tahun	106,682	124,163	121,377	117,345	
3 tahun	115,650	136,992	134,375	125,680	
4 tahun	121,395	145,689	143,189	130,883	
5 tahun	125,548	152,241	149,825	134,574	

Data yang digunakan merupakan data transformasi, *return level* yang diperoleh harus ditransformasi kembali dari unit margin Frechet ke unit GEV, diperoleh hasil pada Tabel 4.11. Nilai ini selanjutnya dihitung selisihnya dengan nilai aktual data testing yang tertera pada Tabel 4.12 untuk diperoleh nilai RMSE dan MAPE. Nilai RMSE diperoleh adalah sebesar 32,078 dan MAPE diperoleh

27,165%. Nilai RMSE ini apa bila dibandingkan dengan nilai RMSE tanpa transformasi dari unit frechet ke unit GEV memberikan perbedaan yang jauh, yaitu sebesar 102,212. *Return level* melibatkan transformasi dua kali, jauh lebih baik.

Tabel 4.12 Nilai Ekstrem Aktual 2010-2015

Periode	**Nilai Ekstrem Aktual**				
	Kendal	**Legundi**	**Gentong**	**Paron**	**Gemarang**
1 tahun	156	84	75	135	82
2 tahun	156	84	79	135	82
3 tahun	156	84	100	135	95
4 tahun	156	84	100	135	95
5 tahun	156	85	100	190	95

Periode	**Nilai Ekstrem Aktual**			
	Kricak	**Widodaren**	**Kedungharjo**	**Guyung**
1 tahun	83	94	87	124
2 tahun	83	94	126	124
3 tahun	83	94	126	128
4 tahun	83	94	126	130
5 tahun	89	98	126	130

Perhitungan *return level* penelitian ini dilakukan untuk 5 tahun yang terbagi dalam tahun pertama, kedua, ketiga, keempat, dan kelima. Tahun pertama melibatkan 4 blok sehingga nilai T = 4, tahun kedua hingga kelima berturut-turut menggunakan T = 8, T = 12, T = 16, T = 20. Periode estimasi tidak dilakukan hanya pada satu periode agar dapat dibandingkan antara nilai *return level* yang muncul dengan probabilitas yang ada. Probabilitas dari *return level* yang diperoleh adalah 1/T. Perhitungan *return level* bergantung pada seberapa panjang periode yang ingin diprediksi nilai *return level*nya, dan seberapa besar peluang terjadinya kejadian ekstrem yang masih diperhitungkan oleh peneliti.

Penelitian ini menggunakan periode prediksi terbanyak adalah 5 tahun (20 blok periode). Penentuan periode (T) maksimum mempertimbangkan probabilitas yang diperoleh yaitu sebesar

$$\frac{\mathbf{1}}{\mathbf{T}} = \frac{\mathbf{1}}{\mathbf{20}} = 0{,}05 = 5\%$$

Ini adalah peluang maksimum yang dapat peneliti perhitungkan. Prediksi menggunakan nilai T sangat besar menghasilkan *return level* dengan peluang kejadian yang sangat kecil dan tidak berarti. Begitu pula penggunaan nilai T yang sangat kecil menyebabkan kemungkinan terjadinya kejadian ekstrem yang lebih besar dengan peluang yang cukup signifikan menjadi terabaikan. Terlebih lagi berapapun nilai *return level* yang diperoleh memiliki peluang terjadi pada blok periode manapun sejauh masih di dalam periode estimasi. Sebagai contoh dari hasil penelitian ini, pada pos hujan Kendal dengan periode 5 tahun, diperoleh *return level* 134,916 mm/hari dengan peluang kejadian 5%. Apabila perhitungan *return level* hanya dilakukan untuk periode satu tahun, sehingga diperoleh *return level* yang lebih kecil yaitu 100,759 mm/hari, cukup berbahaya karena curah hujan sebesar 134,916 mm/hari yang dapat terjadi antara tahun pertama hingga tahun kelima tidak diperhitungkan/terabaikan.

BAB V
KESIMPULAN DAN SARAN

5.1 Kesimpulan

Berdasarkan penelitian yang telah dilakukan, dapat disimpulkan bahwa estimasi parameter distribusi GEV menggunakan MLE dan estimasi parameter model Smith MSP menggunakan MPLE menghasilkan persamaan yang tidak *closed form,* sehingga harus dilanjutkan menggunakan metode iterasi numerik yaitu BFGS *Quasi Newton* yang memiliki kelebihan lebih cepat mencapai konvergensi dibandingkan metode iterasi numerik lainnya. Kekurangan dari metode numerik ini adalah secara komputasi iterasi melibatkan banyak lokasi kurang praktis.

Perhitungan *return level* (curah hujan ekstrem prediksi) dapat dihitung apabila estimator $\hat{\mu}$, $\hat{\sigma}$, $\hat{\xi}$ diketahui. Ketiga estimator tersebut dapat diperoleh dari suatu model yang disebut model *trend surface.* Pengolahan data curah hujan ekstrem Kabupaten Ngawi menghasilkan model *trend surface* $\hat{\mu}(s) = 2{,}794 + 0{,}242\ v(s)$; $\hat{\sigma}(s) = 1{,}820 + 0{,}111\ v(s)$; $\hat{\xi}(s) = 1{,}012$ dengan nilai RMSE sebesar 32,078 dan nilai MAPE sebesar 27,165%.

5.2 Saran

Saran yang dapat diberikan untuk penelitian mendatang adalah perlu dilakukan perbandingan antara metode iterasi numerik BFGS *Quasi Newton* dengan metode numerik lainnya seperti Nelder-Mead. Perbandingan dari sisi validitas yaitu didasarkan pada nilai RMSE dan MAPE, dan dari sisi kebaikan model yaitu nilai TIC yang dihasilkan.

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Lampiran 1 Fungsi *Likelihood* Model Smith

$$L_p\left(\boldsymbol{\beta}\right)=\prod_{i=1}^{n}\prod_{j=1}^{m-1}\prod_{k=j+1}^{m} f\left(z_{ji},z_{ki};\boldsymbol{\beta}\right)$$

$$=\prod_{i=1}^{n}\prod_{j=1}^{m-1}\prod_{k=j+1}^{m}\left\{\left[\exp\left(-\frac{\Phi(w)}{z_{ji}}-\frac{\Phi(v)}{z_{ki}}\right)\right]\times\right.$$

$$\left.\left(\left[\frac{\Phi(w)}{z_{ji}{}^{2}}+\frac{\varphi(w)}{a(h_{j,k})z_{ji}{}^{2}}-\frac{\varphi(v)}{a(h_{j,k})z_{ji}z_{ki}}\right]\left[\frac{\Phi(v)}{z_{ki}{}^{2}}+\frac{\varphi(v)}{a(h_{j,k})z_{ki}{}^{2}}-\frac{\varphi(w)}{a(h_{j,k})z_{ji}z_{ki}}\right]+\left[\frac{v\varphi(w)}{a(h_{j,k})^{2}z_{ji}{}^{2}z_{ki}}-\frac{w\varphi(v)}{a(h_{j,k})^{2}z_{ji}z_{ki}{}^{2}}\right]\right)\right\}$$

$$=\left[\exp\sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}\left(-\frac{\Phi(w)}{z_{ji}}-\frac{\Phi(v)}{z_{ki}}\right)\right]\times$$

$$\prod_{i=1}^{n}\prod_{j=1}^{m-1}\prod_{k=j+1}^{m}\left\{\left(\left[\frac{\Phi(w)}{z_{ji}{}^{2}}+\frac{\varphi(w)}{a(h_{j,k})z_{ji}{}^{2}}-\frac{\varphi(v)}{a(h_{j,k})z_{ji}z_{ki}}\right]\left[\frac{\Phi(v)}{z_{ki}{}^{2}}+\frac{\varphi(v)}{a(h_{j,k})z_{ki}{}^{2}}-\frac{\varphi(w)}{a(h_{j,k})z_{ji}z_{ki}}\right]+\left[\frac{v\varphi(w)}{a(h_{j,k})^{2}z_{ji}{}^{2}z_{ki}}-\frac{w\varphi(v)}{a(h_{j,k})^{2}z_{ji}z_{ki}{}^{2}}\right]\right)\right\}$$

Lampiran 2 Fungsi *Ln Likelihood* Model Smith (1)

$$\ell_p(\boldsymbol{\beta}) = \ln\left[L_p\left(\boldsymbol{\beta}\right)\right]$$

$$= \ln\left\{\left[\exp\sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}\left(-\frac{\Phi(w)}{z_{ji}}-\frac{\Phi(v)}{z_{ki}}\right)\right]\times\right.$$

$$\left.\prod_{i=1}^{n}\prod_{j=1}^{m-1}\prod_{k=j+1}^{m}\left\{\left(\left[\frac{\Phi(w)}{z_{ji}^{\ 2}}+\frac{\varphi(w)}{a(h_{j,k})z_{ji}^{\ 2}}-\frac{\varphi(v)}{a(h)z_{ji}z_{ki}}\right]\left[\frac{\Phi(v)}{z_{ki}^{\ 2}}+\frac{\varphi(v)}{a(h_{j,k})z_{ki}^{\ 2}}-\frac{\varphi(w)}{a(h_{j,k})z_{ji}z_{ki}}\right]+\left[\frac{v\varphi(w)}{a(h_{j,k})^2 z_{ji}^{\ 2}z_{ki}}-\frac{w\varphi(v)}{a(h_{j,k})^2 z_{ji}z_{ki}^{\ 2}}\right]\right)\right\}\right\}$$

$$=\left[\sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}\left(-\frac{\Phi(w)}{z_{ji}}-\frac{\Phi(v)}{z_{ki}}\right)\right]\times$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}\ln\left\{\left(\left[\frac{\Phi(w)}{z_{ji}^{\ 2}}+\frac{\varphi(w)}{a(h_{j,k})z_{ji}^{\ 2}}-\frac{\varphi(v)}{a(h_{j,k})z_{ji}z_{ki}}\right]\left[\frac{\Phi(v)}{z_{ki}^{\ 2}}+\frac{\varphi(v)}{a(h_{j,k})z_{ki}^{\ 2}}-\frac{\varphi(w)}{a(h_{j,k})z_{ji}z_{ki}}\right]+\left[\frac{v\varphi(w)}{a(h_{j,k})^2 z_{ji}^{\ 2}z_{ki}}-\frac{w\varphi(v)}{a(h_{j,k})^2 z_{ji}z_{ki}^{\ 2}}\right]\right)\right\}$$

Lampiran 3 Fungsi *Ln Likelihood* Model Smith (2)

$$\ell_p(\boldsymbol{\beta}) = \left[\sum_{i=1}^{n} \sum_{j=1}^{m-1} \sum_{k=j+1}^{m} \left(-\frac{\Phi(w)}{\left(1+\dfrac{\xi_j (x_{ji}-\mu_j)}{\sigma_j}\right)^{1/\xi_j}} - \frac{\Phi(v)}{\left(1+\dfrac{\xi_k (x_{ki}-\mu_k)}{\sigma_k}\right)^{1/\xi_k}} \right) \right] \times$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m-1} \sum_{k=j+1}^{m} \ln \left\{ \left(\left[\frac{\Phi(w)}{\left(1+\dfrac{\xi_j (x_{ji}-\mu_j)}{\sigma_j}\right)^{2/\xi_j}} + \frac{\varphi(w)}{a(h)\left(1+\dfrac{\xi_j (x_{ji}-\mu_j)}{\sigma_j}\right)^{2/\xi_j}} - \frac{\varphi(v)}{a(h_{j,k})\left(1+\dfrac{\xi_j (x_{ji}-\mu_j)}{\sigma_j}\right)^{1/\xi_j}\left(1+\dfrac{\xi_k (x_{ki}-\mu_k)}{\sigma_k}\right)^{1/\xi_k}} \right] \times \right.\right.$$

$$\left[\frac{\Phi(v)}{\left(1+\dfrac{\xi_k (x_{ki}-\mu_k)}{\sigma_k}\right)^{2/\xi_k}} + \frac{\varphi(v)}{a(h_{j,k})\left(1+\dfrac{\xi_k (x_{ki}-\mu_k)}{\sigma_k}\right)^{2/\xi_k}} - \frac{\varphi(w)}{a(h_{j,k})\left(1+\dfrac{\xi_j (x_{ji}-\mu_j)}{\sigma_j}\right)^{1/\xi_j}\left(1+\dfrac{\xi_k (x_{ki}-\mu_k)}{\sigma_k}\right)^{1/\xi_k}} \right] +$$

$$\left.\left. \left[\frac{v\varphi(w)}{a(h_{j,k})^2\left(1+\dfrac{\xi_j (x_{ji}-\mu_j)}{\sigma_j}\right)^{2/\xi_j}\left(1+\dfrac{\xi_k (x_{ki}-\mu_k)}{\sigma_k}\right)^{1/\xi_k}} - \frac{w\varphi(v)}{a(h_{j,k})^2\left(1+\dfrac{\xi_j (x_{ji}-\mu_j)}{\sigma_j}\right)^{1/\xi_j}\left(1+\dfrac{\xi_k (x_{ki}-\mu_k)}{\sigma_k}\right)^{2/\xi_k}} \right] \right) \right\}$$

Lampiran 4 Fungsi *Ln Likelihood* Model Smith (3)

$$\ell_p(\boldsymbol{\beta}_\mu, \boldsymbol{\beta}_\sigma, \boldsymbol{\beta}_\xi) = \left[\sum_{i=1}^{n} \sum_{j=1}^{m-1} \sum_{k=j+1}^{m} \left(-\frac{\Phi(w)}{\left(1 + d_j{}^T \boldsymbol{\beta}_\xi \dfrac{x_{ji} - d_j{}^T \boldsymbol{\beta}_\mu}{d_j{}^T \boldsymbol{\beta}_\sigma}\right)^{1/d_j{}^T \boldsymbol{\beta}_\xi}} - \frac{\Phi(v)}{\left(1 + d_k{}^T \boldsymbol{\beta}_\xi \dfrac{x_{ki} - d_k{}^T \boldsymbol{\beta}_\mu}{d_k{}^T \boldsymbol{\beta}_\sigma}\right)^{1/d_k{}^T \boldsymbol{\beta}_\xi}} \right) \right] \times$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m-1} \sum_{k=j+1}^{m} \ln \left\{ \left(\left(\left[\frac{\Phi(w)}{\left(1 + d_j{}^T \boldsymbol{\beta}_\xi \dfrac{x_{ji} - d_j{}^T \boldsymbol{\beta}_\mu}{d_j{}^T \boldsymbol{\beta}_\sigma}\right)^{2/d_j{}^T \boldsymbol{\beta}_\xi}} + \frac{\varphi(w)}{a(h_{j,k})\left(1 + d_j{}^T \boldsymbol{\beta}_\xi \dfrac{x_{ji} - d_j{}^T \boldsymbol{\beta}_\mu}{d_j{}^T \boldsymbol{\beta}_\sigma}\right)^{2/d_j{}^T \boldsymbol{\beta}_\xi}} - \frac{\varphi(v)}{a(h_{j,k})\left(1 + d_j{}^T \boldsymbol{\beta}_\xi \dfrac{x_{ji} - d_j{}^T \boldsymbol{\beta}_\mu}{d_j{}^T \boldsymbol{\beta}_\sigma}\right)^{1/d_j{}^T \boldsymbol{\beta}_\xi} \left(1 + d_k{}^T \boldsymbol{\beta}_\xi \dfrac{x_{ki} - d_k{}^T \boldsymbol{\beta}_\mu}{d_k{}^T \boldsymbol{\beta}_\sigma}\right)^{1/d_k{}^T \boldsymbol{\beta}_\xi}} \right] \times \right.\right.\right.$$

$$\left[\frac{\Phi(v)}{\left(1+d_k{}^T\boldsymbol{\beta}_\xi\,\frac{x_{ki}-d_k{}^T\boldsymbol{\beta}_\mu}{d_k{}^T\boldsymbol{\beta}_\sigma}\right)^{2/\mathrm{d}_k{}^T\boldsymbol{\beta}_\xi}}+\frac{\varphi(v)}{a(h_{j,k})\left(1+d_k{}^T\boldsymbol{\beta}_\xi\,\frac{x_{ki}-d_k{}^T\boldsymbol{\beta}_\mu}{d_k{}^T\boldsymbol{\beta}_\sigma}\right)^{2/d_j{}^T\boldsymbol{\beta}_\xi}}-\frac{\varphi(w)}{a(h_{j,k})\left(1+d_j{}^T\boldsymbol{\beta}_\xi\,\frac{x_{ji}-d_j{}^T\boldsymbol{\beta}_\mu}{d_j{}^T\boldsymbol{\beta}_\sigma}\right)^{1/d_j{}^T\boldsymbol{\beta}_\xi}\left(1+d_k{}^T\boldsymbol{\beta}_\xi\,\frac{x_{ki}-d_k{}^T\boldsymbol{\beta}_\mu}{d_k{}^T\boldsymbol{\beta}_\sigma}\right)^{1/d_j{}^T\boldsymbol{\beta}_\xi}}\right]+$$

$$\left.\left.\left.\left[\frac{v\varphi(w)}{a(h_{j,k})^2\left(1+d_j{}^T\boldsymbol{\beta}_\xi\,\frac{x_{ji}-d_j{}^T\boldsymbol{\beta}_\mu}{d_j{}^T\boldsymbol{\beta}_\sigma}\right)^{2/d_j{}^T\boldsymbol{\beta}_\xi}\left(1+d_k{}^T\boldsymbol{\beta}_\xi\,\frac{x_{ki}-d_k{}^T\boldsymbol{\beta}_\mu}{d_k{}^T\boldsymbol{\beta}_\sigma}\right)^{1/d_j{}^T\boldsymbol{\beta}_\xi}}-\frac{w\varphi(v)}{a(h_{j,k})^2\left(1+d_j{}^T\boldsymbol{\beta}_\xi\,\frac{x_{ji}-d_j{}^T\boldsymbol{\beta}_\mu}{d_j{}^T\boldsymbol{\beta}_\sigma}\right)^{1/d_j{}^T\boldsymbol{\beta}_\xi}\left(1+d_k{}^T\boldsymbol{\beta}_\xi\,\frac{x_{ki}-d_k{}^T\boldsymbol{\beta}_\mu}{d_k{}^T\boldsymbol{\beta}_\sigma}\right)^{2/d_j{}^T\boldsymbol{\beta}_\xi}}\right]\right]\right)\right\}$$

Lampiran 5 Turunan Pertama Fungsi *Ln Likelihood* Model Smith Terhadap Parameter $\boldsymbol{\beta_\mu}$

$$
\frac{\partial \ell(\boldsymbol{\beta})}{\partial \beta_\mu}=\frac{\begin{array}{l}
\partial\left\{\left[\displaystyle\sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}\left(-\frac{\Phi(w)}{\left(1+d_j{}^T\boldsymbol{\beta}_\xi\,\frac{x_{ji}-d_j{}^T\boldsymbol{\beta}_\mu}{d_j{}^T\boldsymbol{\beta}_\sigma}\right)^{1/d_j{}^T\boldsymbol{\beta}_\xi}}-\frac{\Phi(v)}{\left(1+d_k{}^T\boldsymbol{\beta}_\xi\,\frac{x_{ki}-d_k{}^T\boldsymbol{\beta}_\mu}{d_k{}^T\boldsymbol{\beta}_\sigma}\right)^{1/d_k{}^T\boldsymbol{\beta}_\xi}}\right)\right]\times\right.\\[3ex]
\displaystyle\sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}\ln\left\{\left(\left(\left[\frac{\Phi(w)}{\left(1+d_j{}^T\boldsymbol{\beta}_\xi\,\frac{x_{ji}-d_j{}^T\boldsymbol{\beta}_\mu}{d_j{}^T\boldsymbol{\beta}_\sigma}\right)^{2/d_j{}^T\boldsymbol{\beta}_\xi}}+\frac{\varphi(w)}{a(h_{j,k})\left(1+d_j{}^T\boldsymbol{\beta}_\xi\,\frac{x_{ji}-d_j{}^T\boldsymbol{\beta}_\mu}{d_j{}^T\boldsymbol{\beta}_\sigma}\right)^{2/d_j{}^T\boldsymbol{\beta}_\xi}}-\frac{\varphi(v)}{a(h_{j,k})\left(1+d_j{}^T\boldsymbol{\beta}_\xi\,\frac{x_{ji}-d_j{}^T\boldsymbol{\beta}_\mu}{d_j{}^T\boldsymbol{\beta}_\sigma}\right)^{1/d_j{}^T\boldsymbol{\beta}_\xi}\left(1+d_k{}^T\boldsymbol{\beta}_\xi\,\frac{x_{ki}-d_k{}^T\boldsymbol{\beta}_\mu}{d_k{}^T\boldsymbol{\beta}_\sigma}\right)^{1/d_k{}^T\boldsymbol{\beta}_\xi}}\right]\times\right.\right.\right.\\[3ex]
\left[\dfrac{\Phi(v)}{\left(1+d_k{}^T\boldsymbol{\beta}_\xi\,\frac{x_{ki}-d_k{}^T\boldsymbol{\beta}_\mu}{d_k{}^T\boldsymbol{\beta}_\sigma}\right)^{2/d_k{}^T\boldsymbol{\beta}_\xi}}+\dfrac{\varphi(v)}{a(h_{j,k})\left(1+d_k{}^T\boldsymbol{\beta}_\xi\,\frac{x_{ki}-d_k{}^T\boldsymbol{\beta}_\mu}{d_k{}^T\boldsymbol{\beta}_\sigma}\right)^{2/d_j{}^T\boldsymbol{\beta}_\xi}}-\dfrac{\varphi(w)}{a(h_{j,k})\left(1+d_j{}^T\boldsymbol{\beta}_\xi\,\frac{x_{ji}-d_j{}^T\boldsymbol{\beta}_\mu}{d_j{}^T\boldsymbol{\beta}_\sigma}\right)^{1/d_j{}^T\boldsymbol{\beta}_\xi}\left(1+d_k{}^T\boldsymbol{\beta}_\xi\,\frac{x_{ki}-d_k{}^T\boldsymbol{\beta}_\mu}{d_k{}^T\boldsymbol{\beta}_\sigma}\right)^{1/d_j{}^T\boldsymbol{\beta}_\xi}}\right]+\\[3ex]
\left.\left.\left.\left.\left.\left[\dfrac{v\varphi(w)}{a(h_{j,k})^2\left(1+d_j{}^T\boldsymbol{\beta}_\xi\,\frac{x_{ji}-d_j{}^T\boldsymbol{\beta}_\mu}{d_j{}^T\boldsymbol{\beta}_\sigma}\right)^{2/d_j{}^T\boldsymbol{\beta}_\xi}\left(1+d_k{}^T\boldsymbol{\beta}_\xi\,\frac{x_{ki}-d_k{}^T\boldsymbol{\beta}_\mu}{d_k{}^T\boldsymbol{\beta}_\sigma}\right)^{1/d_j{}^T\boldsymbol{\beta}_\xi}}-\dfrac{w\varphi(v)}{a(h_{j,k})^2\left(1+d_j{}^T\boldsymbol{\beta}_\xi\,\frac{x_{ji}-d_j{}^T\boldsymbol{\beta}_\mu}{d_j{}^T\boldsymbol{\beta}_\sigma}\right)^{1/d_j{}^T\boldsymbol{\beta}_\xi}\left(1+d_k{}^T\boldsymbol{\beta}_\xi\,\frac{x_{ki}-d_k{}^T\boldsymbol{\beta}_\mu}{d_k{}^T\boldsymbol{\beta}_\sigma}\right)^{2/d_j{}^T\boldsymbol{\beta}_\xi}}\right]\right]\right)\right)\right\}\right\}
\end{array}}{\partial \beta_\mu}
$$

$$\frac{\partial \ell(\boldsymbol{\beta})}{\partial \beta_\mu} = 0$$

$$0 = \sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}\left(-\frac{\Phi(w)\left[d_j\beta_\xi \left(\frac{d_j}{d_j\beta_\sigma} \right) \right]}{\left[1 + d_j\beta_\xi \left(\frac{x_{ji} - d_j\beta_\mu}{d_j\beta_\sigma} \right) \right]^{\frac{1}{d_j\beta_\xi}} d_j\beta_\xi \left[1 + d_j\beta_\xi \left(\frac{x_{ji} - d_j\beta_\mu}{d_j\beta_\sigma} \right) \right]} - \frac{\Phi(v)\left[d_k\beta_\xi \left(\frac{d_k}{d_k\beta_\sigma} \right) \right]}{\left[1 + d_k\beta_\xi \left(\frac{x_{ki} - d_k\beta_\mu}{d_k\beta_\sigma} \right) \right]^{\frac{1}{d_k\beta_\xi}} d_k\beta_\xi \left[1 + d_k\beta_\xi \frac{x_{ki} - d_k\beta_\mu}{d_k\beta_\sigma} \right]} \right) \times$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m} \ln \left(\left(\frac{\Phi(w)}{\left[1 + d_j\beta_\xi \left(\frac{x_{ji} - d_j\beta_\mu}{d_j\beta_\sigma} \right) \right]^{\frac{2}{d_j\beta_\xi}}} + \frac{\varphi(w)}{a(h_{j,k})\left[1 + d_j\beta_\xi \left(\frac{x_{ji} - d_j\beta_\mu}{d_j\beta_\sigma} \right) \right]^{\frac{2}{d_j\beta_\xi}}} - \frac{\varphi(v)}{a(h_{j,k})\left[1 + d_j\beta_\xi \left(\frac{x_{ji} - d_j\beta_\mu}{d_j\beta_\sigma} \right) \right]^{\frac{1}{d_j\beta_\xi}} \left[1 + d_k\beta_\xi \left(\frac{x_{ki} - d_k\beta_\mu}{d_k\beta_\sigma} \right) \right]^{\frac{1}{d_k\beta_\xi}}} \right) \times \right.$$

$$\left. \left(\frac{\Phi(v)}{\left[1 + d_k\beta_\xi \left(\frac{x_{ki} - d_k\beta_\mu}{d_k\beta_\sigma} \right) \right]^{\frac{2}{d_k\beta_\xi}}} + \frac{\varphi(v)}{a(h_{j,k})\left[1 + d_k\beta_\xi \left(\frac{x_{ki} - d_k\beta_\mu}{d_k\beta_\sigma} \right) \right]^{\frac{2}{d_j\beta_\xi}}} - \frac{\varphi(w)}{a(h_{j,k})\left[1 + d_j\beta_\xi \left(\frac{x_{ji} - d_j\beta_\mu}{d_j\beta_\sigma} \right) \right]^{\frac{1}{d_j\beta_\xi}} \left[1 + d_k\beta_\xi \left(\frac{x_{ki} - d_k\beta_\mu}{d_k\beta_\sigma} \right) \right]^{\frac{1}{d_j\beta_\xi}}} \right) + \right.$$

$$
\left.\frac{v\varphi(w)}{a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\dfrac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\dfrac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}-\frac{w\varphi(v)}{a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\dfrac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\dfrac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}\right)+
$$

$$
\left(-\frac{\Phi(w)}{\left[1+d_j\beta_\xi\left(\dfrac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}-\frac{\Phi(v)}{\left[1+d_k\beta_\xi\left(\dfrac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}}\right)\Bigg(\Bigg(\Bigg(\frac{2\Phi(w)\left(d_j\beta_\xi\dfrac{d_j}{d_j\beta_\sigma}\right)}{\left[1+d_j\beta_\xi\left(\dfrac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}d_j\beta_\xi\left[1+d_j\beta_\xi\left(\dfrac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}+
$$

$$\frac{2\varphi(w)\left[a(h_{j,k})\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}} d_j\beta_\xi\frac{d_j}{d_j\beta_\sigma}}{d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\right]}{\left\{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right\}^2}+\left[\varphi(v)\left(-a(h_{j,k})\left[\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}} d_j\beta_\xi\frac{d_j}{d_j\beta_\sigma}}{d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right]\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}-\right.\right.$$

$$a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)}{d_k\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right]\Bigg)\Bigg]\Bigg/\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}\right)^2\Bigg)\times$$

$$\left(\frac{\Phi(v)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}+\frac{\varphi(v)}{a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}\right)+$$

$$\left(\frac{\Phi(w)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}+\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(v)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}}\right)\times$$

$$\left(\frac{2\Phi(v)d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}d_k\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}+\frac{2\varphi(v)a(h_{j,k})}{\left(a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^2}\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}+\right.$$

$$\left(\varphi(w)\left(-a(h_{j,k})\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}d_j\beta_\xi\frac{d_j}{d_j\beta_\sigma}}{d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}-a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}d_k\beta_\xi\frac{d_k}{d_k\beta_\sigma}}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\right)\right)\Bigg/$$

$$\left.\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\right)^2\right)\right)-\left(\left(v\varphi(w)\left(-2\left(a(h_{j,k})^2\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)}{d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}\right)-\right.\right.\right.\right.$$

$$a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}d_k\beta_\xi\frac{d_k}{d_k\beta_\sigma}}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\Bigg)\Bigg)\Bigg/\left(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\right)^2+$$

$$\Bigg(w\varphi(v)\Bigg(-(a(h_{j,k})^2)\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)}{d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}-2\Bigg((a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\times$$

$$\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}d_k\beta_\xi\frac{d_k}{d_k\beta_\sigma}}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\Bigg)\Bigg)\Bigg)\Bigg/\Bigg(\Bigg(\left(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}\right)^2\Bigg)\Bigg)\Bigg)/$$

$$\left(\left(\left(\left(\frac{\Phi(w)}{\left[1+d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}+\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(v)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}}\right)\times\right.\right.\right.$$

$$\left(\frac{\Phi(v)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}}+\frac{\varphi(v)}{a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}\right)+$$

$$\left.\left.\frac{v\varphi(w)}{a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}-\frac{w\varphi(v)}{a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}\right)\right)$$

Lampiran 6 Turunan Pertama Fungsi *Ln Likelihood* Model Smith Terhadap Parameter $\boldsymbol{\beta}_{\sigma}$

$$
\frac{\partial \ell(\boldsymbol{\beta})}{\partial \beta_{\sigma}} = \frac{\begin{aligned}
&\partial\Bigg\{\Bigg[\sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}\Bigg(-\frac{\Phi(w)}{\left(1+d_j^{T}\boldsymbol{\beta}_{\xi}\frac{x_{ji}-d_j^{T}\boldsymbol{\beta}_{\mu}}{d_j^{T}\boldsymbol{\beta}_{\sigma}}\right)^{1/d_j^{T}\boldsymbol{\beta}_{\xi}}}-\frac{\Phi(v)}{\left(1+d_k^{T}\boldsymbol{\beta}_{\xi}\frac{x_{ki}-d_k^{T}\boldsymbol{\beta}_{\mu}}{d_k^{T}\boldsymbol{\beta}_{\sigma}}\right)^{1/d_k^{T}\boldsymbol{\beta}_{\xi}}}\Bigg)\Bigg]\times \\
&\sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}\ln\Bigg\{\Bigg(\Bigg(\Bigg[\frac{\Phi(w)}{\left(1+d_j^{T}\boldsymbol{\beta}_{\xi}\frac{x_{ji}-d_j^{T}\boldsymbol{\beta}_{\mu}}{d_j^{T}\boldsymbol{\beta}_{\sigma}}\right)^{2/d_j^{T}\boldsymbol{\beta}_{\xi}}}+\frac{\varphi(w)}{a(h_{j,k})\left(1+d_j^{T}\boldsymbol{\beta}_{\xi}\frac{x_{ji}-d_j^{T}\boldsymbol{\beta}_{\mu}}{d_j^{T}\boldsymbol{\beta}_{\sigma}}\right)^{2/d_j^{T}\boldsymbol{\beta}_{\xi}}}-\frac{\varphi(v)}{a(h_{j,k})\left(1+d_j^{T}\boldsymbol{\beta}_{\xi}\frac{x_{ji}-d_j^{T}\boldsymbol{\beta}_{\mu}}{d_j^{T}\boldsymbol{\beta}_{\sigma}}\right)^{1/d_j^{T}\boldsymbol{\beta}_{\xi}}\left(1+d_k^{T}\boldsymbol{\beta}_{\xi}\frac{x_{ki}-d_k^{T}\boldsymbol{\beta}_{\mu}}{d_k^{T}\boldsymbol{\beta}_{\sigma}}\right)^{1/d_k^{T}\boldsymbol{\beta}_{\xi}}}\Bigg]\times \\
&\Bigg[\frac{\Phi(v)}{\left(1+d_k^{T}\boldsymbol{\beta}_{\xi}\frac{x_{ki}-d_k^{T}\boldsymbol{\beta}_{\mu}}{d_k^{T}\boldsymbol{\beta}_{\sigma}}\right)^{2/d_k^{T}\boldsymbol{\beta}_{\xi}}}+\frac{\varphi(v)}{a(h_{j,k})\left(1+d_k^{T}\boldsymbol{\beta}_{\xi}\frac{x_{ki}-d_k^{T}\boldsymbol{\beta}_{\mu}}{d_k^{T}\boldsymbol{\beta}_{\sigma}}\right)^{2/d_j^{T}\boldsymbol{\beta}_{\xi}}}-\frac{\varphi(w)}{a(h_{j,k})\left(1+d_j^{T}\boldsymbol{\beta}_{\xi}\frac{x_{ji}-d_j^{T}\boldsymbol{\beta}_{\mu}}{d_j^{T}\boldsymbol{\beta}_{\sigma}}\right)^{1/d_j^{T}\boldsymbol{\beta}_{\xi}}\left(1+d_k^{T}\boldsymbol{\beta}_{\xi}\frac{x_{ki}-d_k^{T}\boldsymbol{\beta}_{\mu}}{d_k^{T}\boldsymbol{\beta}_{\sigma}}\right)^{1/d_j^{T}\boldsymbol{\beta}_{\xi}}}\Bigg]\times \\
&+\Bigg[\frac{v\varphi(w)}{a(h_{j,k})^{2}\left(1+d_j^{T}\boldsymbol{\beta}_{\xi}\frac{x_{ji}-d_j^{T}\boldsymbol{\beta}_{\mu}}{d_j^{T}\boldsymbol{\beta}_{\sigma}}\right)^{2/d_j^{T}\boldsymbol{\beta}_{\xi}}\left(1+d_k^{T}\boldsymbol{\beta}_{\xi}\frac{x_{ki}-d_k^{T}\boldsymbol{\beta}_{\mu}}{d_k^{T}\boldsymbol{\beta}_{\sigma}}\right)^{1/d_j^{T}\boldsymbol{\beta}_{\xi}}}-\frac{w\varphi(v)}{a(h_{j,k})^{2}\left(1+d_j^{T}\boldsymbol{\beta}_{\xi}\frac{x_{ji}-d_j^{T}\boldsymbol{\beta}_{\mu}}{d_j^{T}\boldsymbol{\beta}_{\sigma}}\right)^{1/d_j^{T}\boldsymbol{\beta}_{\xi}}\left(1+d_k^{T}\boldsymbol{\beta}_{\xi}\frac{x_{ki}-d_k^{T}\boldsymbol{\beta}_{\mu}}{d_k^{T}\boldsymbol{\beta}_{\sigma}}\right)^{2/d_j^{T}\boldsymbol{\beta}_{\xi}}}\Bigg]\Bigg)\Bigg)\Bigg\}\Bigg\}
\end{aligned}}{\partial \beta_{\sigma}}
$$

$$\frac{\partial \ell(\boldsymbol{\beta})}{\partial \beta_\sigma} = 0$$

$$0 = \sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}\left(-\frac{\Phi(w)\left[d_j\beta_\xi \left(\frac{(x_{ji}-d_j\beta_\mu)d_j}{(d_j\beta_\sigma)^2} \right)\right]}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}} d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]} - \frac{\Phi(v)\left[d_k\beta_\xi \left(\frac{(x_{ki}-d_k\beta_\mu)d_k}{(d_k\beta_\sigma)^2} \right)\right]}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}} d_k\beta_\xi\left[1+d_k\beta_\xi\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right]} \right)\times$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}\ln\left(\left(\frac{1}{d_j\beta_\sigma d_k\beta_\sigma}\left(\left(\frac{\Phi(w)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}} + \frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}} - \frac{\varphi(v)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}}\right)\times\right.\right.\right.$$

$$\left(\frac{\Phi(v)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}} + \frac{\varphi(v)}{a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}} - \frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}\right)+$$

$$\frac{v\varphi(w)}{a(h_{j,k})^2\Bigg[1+d_j\beta_\xi\Bigg(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\Bigg)\Bigg]^{\frac{2}{d_j\beta_\xi}}\Bigg[1+d_k\beta_\xi\Bigg(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\Bigg)\Bigg]^{\frac{1}{d_j\beta_\xi}}}-\frac{w\varphi(v)}{a(h_{j,k})^2\Bigg[1+d_j\beta_\xi\Bigg(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\Bigg)\Bigg]^{\frac{1}{d_j\beta_\xi}}\Bigg[1+d_k\beta_\xi\Bigg(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\Bigg)\Bigg]^{\frac{2}{d_j\beta_\xi}}}\Bigg)\Bigg)+$$

$$\Bigg(-\frac{\Phi(w)}{\Bigg[1+d_j\beta_\xi\Bigg(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\Bigg)\Bigg]^{\frac{1}{d_j\beta_\xi}}}-\frac{\Phi(v)}{\Bigg[1+d_k\beta_\xi\Bigg(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\Bigg)\Bigg]^{\frac{1}{d_k\beta_\xi}}}\Bigg)\Bigg)\Bigg(\Bigg(\Bigg(\Bigg(\Bigg(\frac{2\Phi(w)d_j\beta_\xi\frac{(x_{ji}-d_j\beta_\mu)d_j}{(d_j\beta_\sigma)^2}}{\Bigg[1+d_j\beta_\xi\Bigg(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\Bigg)\Bigg]^{\frac{2}{d_j\beta_\xi}}d_j\beta_\xi\Bigg[1+d_j\beta_\xi\Bigg(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\Bigg)\Bigg]}+$$

$$\frac{2\varphi(w)\left[a(h_{j,k})\left(\frac{\Bigg[1+d_j\beta_\xi\Bigg(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\Bigg)\Bigg]^{\frac{2}{d_j\beta_\xi}}d_j\beta_\xi\frac{(x_{ji}-d_j\beta_\mu)d_j}{(d_j\beta_\sigma)^2}}{d_j\beta_\xi\Bigg[1+d_j\beta_\xi\Bigg(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\Bigg)\Bigg]}\right)\right]}{\left\{a(h_{j,k})\Bigg[1+d_j\beta_\xi\Bigg(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\Bigg)\Bigg]^{\frac{2}{d_j\beta_\xi}}\right\}^2}+\Bigg[\varphi(v)\Bigg(-a(h_{j,k})\left[\frac{\Bigg[1+d_j\beta_\xi\Bigg(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\Bigg)\Bigg]^{\frac{1}{d_j\beta_\xi}}d_j\beta_\xi\frac{(x_{ji}-d_j\beta_\mu)d_j}{(d_j\beta_\sigma)^2}}{d_j\beta_\xi\Bigg[1+d_j\beta_\xi\Bigg(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\Bigg)\Bigg]}\right]\Bigg[1+d_k\beta_\xi\Bigg(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\Bigg)\Bigg]^{\frac{1}{d_k\beta_\xi}}-$$

$$a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}d_k\beta_\xi\left(\frac{(x_{ki}-d_k\beta_\mu)d_k}{(d_k\beta_\sigma)^2}\right)}{d_k\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right]\Bigg)\Bigg]\Bigg/\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}\right)^2\Bigg)\times$$

$$\left(\frac{\Phi(v)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}}+\frac{\varphi(v)}{a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}\right)+$$

$$\left(\frac{\Phi(w)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}+\frac{\varphi(w)}{a(h)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(v)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}}\right)\times$$

$$\left(\frac{2\Phi(v)\left(d_k\beta_\xi\left(\frac{(x_{ki}-d_k\beta_\mu)d_k}{(d_k\beta_\sigma)^2}\right)\right)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}d_k\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}+\frac{2\varphi(v)a(h)}{\left(a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^2}\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}d_k\beta_\xi\left(\frac{(x_{ki}-d_k\beta_\mu)d_k}{(d_k\beta_\sigma)^2}\right)}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}+\right.$$

$$\left(\varphi(w)\left(-a(h_{j,k})\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}d_j\beta_\xi\frac{(x_{ji}-d_j\beta_\mu)d_j}{(d_j\beta_\sigma)^2}}{d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}-a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\times\right.\right.$$

$$\left.\left.\left.\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}d_k\beta_\xi\frac{\left(\frac{(x_{ki}-d_k\beta_\mu)d_k}{(d_k\beta_\sigma)^2}\right)d_k}{d_k\beta_\sigma}}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\right)\right)\middle/\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\right)^2\right)-$$

$$\Bigg(v\varphi(w) \Bigg(-2 \Bigg(a(h_{j,k})^2 \Bigg(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}} d_j\beta_\xi\left(\frac{(x_{ji}-d_j\beta_\mu)d_j}{(d_j\beta_\sigma)^2}\right)}{d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]} \Bigg) \left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}} \Bigg) - a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}} \times$$

$$\Bigg(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}} d_k\beta_\xi\left(\frac{(x_{ki}-d_k\beta_\mu)d_k}{(d_k\beta_\sigma)^2}\right)}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]} \Bigg)\Bigg)\Bigg)\Bigg) \Bigg/ \Bigg(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}} \left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}} \Bigg)^2 +$$

$$\Bigg(w\varphi(v) \Bigg(-(a(h_{j,k})^2) \Bigg(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}} d_j\beta_\xi\left(\frac{(x_{ji}-d_j\beta_\mu)d_k}{(d_j\beta_\sigma)^2}\right)}{d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]} \Bigg) \left[1+d_k\beta_\xi\left(\frac{x_k-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}} - 2\Bigg((a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}} \times$$

$$\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}} d_k\beta_\xi\left(\frac{(x_{ki}-d_k\beta_\mu)d_k}{(d_k\beta_\sigma)^2}\right)}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\Bigg)\Bigg)\Bigg) \Bigg/ \Bigg(\Bigg(\Bigg(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\Bigg)^2\Bigg)/$$

$$\Bigg(\Bigg(\left(\frac{\Phi(w)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}+\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(v)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}}\right)\times$$

$$\left(\frac{\Phi(v)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{x_k\beta_\xi}}}+\frac{\varphi(v)}{a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}\right)+$$

$$\left.\left.\frac{v\varphi(w)}{a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}-\frac{v\varphi(w)}{a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}\right)\right)$$

Lampiran 7 Turunan Pertama Fungsi *Ln Likelihood* Model Smith Terhadap Parameter $\boldsymbol{\beta}_\xi$

$$\frac{\partial\ell(\boldsymbol{\beta})}{\partial\beta_\xi}=\frac{\partial\left\{\left[\sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}\left(-\frac{\Phi(w)}{\left(1+d_j{}^T\boldsymbol{\beta}_\xi\frac{x_{ji}-d_j{}^T\boldsymbol{\beta}_\mu}{d_j{}^T\boldsymbol{\beta}_\sigma}\right)^{1/d_j{}^T\boldsymbol{\beta}_\xi}}-\frac{\Phi(v)}{\left(1+d_k{}^T\boldsymbol{\beta}_\xi\frac{x_{ki}-d_k{}^T\boldsymbol{\beta}_\mu}{d_k{}^T\boldsymbol{\beta}_\sigma}\right)^{1/d_k{}^T\boldsymbol{\beta}_\xi}}\right)\right]\times\sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}\ln\left\{\left(\left(\left[\frac{\Phi(w)}{\left(1+d_j{}^T\boldsymbol{\beta}_\xi\frac{x_{ji}-d_j{}^T\boldsymbol{\beta}_\mu}{d_j{}^T\boldsymbol{\beta}_\sigma}\right)^{2/d_j{}^T\boldsymbol{\beta}_\xi}}+\frac{\varphi(w)}{a(h_{j,k})\left(1+d_j{}^T\boldsymbol{\beta}_\xi\frac{x_{ji}-d_j{}^T\boldsymbol{\beta}_\mu}{d_j{}^T\boldsymbol{\beta}_\sigma}\right)^{2/d_j{}^T\boldsymbol{\beta}_\xi}}-\frac{\varphi(v)}{a(h_{j,k})\left(1+d_j{}^T\boldsymbol{\beta}_\xi\frac{x_{ji}-d_j{}^T\boldsymbol{\beta}_\mu}{d_j{}^T\boldsymbol{\beta}_\sigma}\right)^{1/d_j{}^T\boldsymbol{\beta}_\xi}\left(1+d_k{}^T\boldsymbol{\beta}_\xi\frac{x_{ki}-d_k{}^T\boldsymbol{\beta}_\mu}{d_k{}^T\boldsymbol{\beta}_\sigma}\right)^{1/d_k{}^T\boldsymbol{\beta}_\xi}}\right]\times\left[\frac{\Phi(v)}{\left(1+d_k{}^T\boldsymbol{\beta}_\xi\frac{x_{ki}-d_k{}^T\boldsymbol{\beta}_\mu}{d_k{}^T\boldsymbol{\beta}_\sigma}\right)^{2/\zeta_k}}+\frac{\varphi(v)}{a(h_{j,k})\left(1+d_k{}^T\boldsymbol{\beta}_\xi\frac{x_{ki}-d_k{}^T\boldsymbol{\beta}_\mu}{d_k{}^T\boldsymbol{\beta}_\sigma}\right)^{2/d_j{}^T\boldsymbol{\beta}_\xi}}-\frac{\varphi(w)}{a(h_{j,k})\left(1+d_j{}^T\boldsymbol{\beta}_\xi\frac{x_{ji}-d_j{}^T\boldsymbol{\beta}_\mu}{d_j{}^T\boldsymbol{\beta}_\sigma}\right)^{1/d_j{}^T\boldsymbol{\beta}_\xi}\left(1+d_k{}^T\boldsymbol{\beta}_\xi\frac{x_{ki}-d_k{}^T\boldsymbol{\beta}_\mu}{d_k{}^T\boldsymbol{\beta}_\sigma}\right)^{1/d_j{}^T\boldsymbol{\beta}_\xi}}\right]+\left[\frac{v\varphi(w)}{a(h_{j,k})^2\left(1+d_j{}^T\boldsymbol{\beta}_\xi\frac{x_{ji}-d_j{}^T\boldsymbol{\beta}_\mu}{d_j{}^T\boldsymbol{\beta}_\sigma}\right)^{2/d_j{}^T\boldsymbol{\beta}_\xi}\left(1+d_k{}^T\boldsymbol{\beta}_\xi\frac{x_{ki}-d_k{}^T\boldsymbol{\beta}_\mu}{d_k{}^T\boldsymbol{\beta}_\sigma}\right)^{1/d_j{}^T\boldsymbol{\beta}_\xi}}-\frac{w\varphi(v)}{a(h_{j,k})^2\left(1+d_j{}^T\boldsymbol{\beta}_\xi\frac{x_{ji}-d_j{}^T\boldsymbol{\beta}_\mu}{d_j{}^T\boldsymbol{\beta}_\sigma}\right)^{1/d_j{}^T\boldsymbol{\beta}_\xi}\left(1+d_k{}^T\boldsymbol{\beta}_\xi\frac{x_{ki}-d_k{}^T\boldsymbol{\beta}_\mu}{d_k{}^T\boldsymbol{\beta}_\sigma}\right)^{2/d_j{}^T\boldsymbol{\beta}_\xi}}\right]\right)\right)\right\}\right\}}{\partial\beta_\xi}$$

$$\frac{\partial \ell(\boldsymbol{\beta})}{\partial \beta_\zeta} = 0$$

$$0 = \sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}\left(\frac{\dfrac{\Phi(w)\left[-d_j \ln\left[1 + d_j\beta_\xi\left(\dfrac{x_{ji} - d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]\right]}{(d_j\beta_\xi)^2} + \dfrac{d_j\left(\dfrac{x_{ji} - d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\xi\left[1 + d_j\beta_\xi\left(\dfrac{x_{ji} - d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}}{d_j\beta_\xi\left[1 + d_j\beta_\xi\left(\dfrac{x_{ji} - d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}} + \frac{\Phi(v)\left[\dfrac{-\ln d_k\left[1 + d_k\beta_\xi\left(\dfrac{x_{ki} - d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}{(d_k\beta_\xi)^2} + \dfrac{d_k\left(\dfrac{(x_{ki} - d_k\beta_\mu)d_k}{(d_k\beta_\sigma)^2}\right)}{d_k\beta_\xi\left[1 + d_k\beta_\xi\dfrac{x_{ki} - d_k\beta_\mu}{d_k\beta_\sigma}\right]}\right]}{\left[1 + d_k\beta_\xi\left(\dfrac{x_{ki} - d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}} \right) \times$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m} \ln\left((((\frac{\Phi(w)}{\left[1 + d_j\beta_\xi\left(\dfrac{x_{ji} - d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}} + \frac{\varphi(w)}{a(h_{j,k})\left[1 + d_j\beta_\xi\left(\dfrac{x_{ji} - d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}} - \frac{\varphi(v)}{a(h_{j,k})\left[1 + d_j\beta_\xi\left(\dfrac{x_{ji} - d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1 + d_k\beta_\xi\left(\dfrac{x_{ki} - d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}} \right) \times$$

$$
\left(\frac{\Phi(v)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}} + \frac{\varphi(v)}{a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}} - \frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}} \right) +
$$

$$
\frac{v\varphi(w)}{a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{x_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}} - \frac{w\varphi(v)}{a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{x_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}} \Bigg)\Bigg) \times
$$

$$
\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}-1}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{x_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}-1} \Bigg) + \left(-\frac{\Phi(w)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}} - \frac{\Phi(v)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}} \right) \times
$$

$$\Bigg(\Bigg(\Bigg(\Bigg(\Bigg(-\frac{-\frac{2\Phi(w)d_j\ln\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}{(d_j\beta_\xi)^2}+\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{1}{\left\{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right\}^2}\times$$

$$\Bigg(w\Bigg(a(h_{j,k})\Bigg(\Bigg(\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(-\frac{\left(-2d_j\ln\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]\right)}{(d_j\beta_\xi)^2}+\frac{2d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\Bigg)\Bigg)\Bigg)\Bigg)+\Bigg[\varphi(v)\Bigg(a(h_{j,k})\Bigg(\left(1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\xi}}\times$$

$$\left(-\frac{d_j\ln\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}{(d_j\beta_\xi)^2}+\frac{d_j\frac{(x_{ji}-d_j\beta_\mu)}{(d_j\beta_\sigma)^2}}{d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\Bigg)\Bigg)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{x_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}+a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\times$$

$$\left(\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}\left(-\frac{d_k\ln\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}{(d_k\beta_\xi)^2}+\frac{d_k\frac{(x_{ki}-d_k\beta_\mu)}{d_k\beta_\sigma}}{d_k\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\right)\right)\right)\Bigg/\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}\right)^2\right)\times$$

$$\left(\frac{\Phi(v)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}}+\frac{\varphi(v)}{a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}\right)+$$

$$\left(\frac{\Phi(w)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}+\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(v)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}}\right)\times$$

$$\Bigg(-\frac{\Phi(v)\left(\frac{-2d_k \ln\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}{(d_k\beta_\xi)^2} + \frac{2\frac{(x_{ki}-d_k\beta_\mu)d_k}{(d_k\beta_\sigma)^2}}{d_k\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]} \right)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}} - \frac{1}{\left(a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^2}\Bigg(\varphi(v)\Bigg(a(h_{j,k})\Bigg(\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\times$$

$$\Bigg(\left(\frac{-2d_j\ln\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}{(d_j\beta_\xi)^2} + \frac{2d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\Bigg)\Bigg)\Bigg)\Bigg) + \Bigg(\varphi(w)\Bigg(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\times$$

$$\left(\left(\frac{d_j \ln\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}{(d_j\beta_\xi)^2}+\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}+\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\times$$

$$\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(\left(\frac{-d_j \ln\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}{(d_j\beta_\xi)^2}+\frac{2d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\right)\Bigg)\Bigg)\Bigg/\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\right)^2\Bigg)-$$

$$\left(v\varphi(w)\left(a(h_{j,k})^2\left(\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)\left(\left(\frac{2d_j \ln\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}{(d_j\beta_\xi)^2}+\frac{2d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}+\right.\right.$$

$$a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(\frac{\left(d_j\ln\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]\right.}{(d_j\beta_\xi)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\right)\Bigg)\Bigg)\Bigg)/$$

$$\left(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\right)^2+\Bigg(w\varphi(v)\Bigg(a(h_{j,k})^2\Bigg(\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\Bigg(\frac{\left(2d_j\ln\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]\right.}{(d_j\beta_\xi)^2}+$$

$$\left.\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\Bigg)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}+a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\Bigg(\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\times$$

$$
\Bigg(\Bigg(\frac{2d_j \ln\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}{(d_j\beta_\xi)^2}+\frac{2d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\Bigg)\Bigg)\Bigg)\Bigg)\Bigg) \Bigg/ \Bigg(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\Bigg)^2\Bigg)\Bigg)\Bigg/
$$

$$
\Bigg(\Bigg(\Bigg(\Bigg(\frac{\Phi(w)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}+\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(v)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}}\Bigg)\times
$$

$$
\Bigg(\frac{\Phi(v)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}}+\frac{\varphi(v)}{a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}\Bigg)+
$$

$$\left.\left.\frac{v\varphi(w)}{a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}-\frac{w\varphi(v)}{a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}\right)\right)$$

Lampiran 8 Data Curah Hujan Harian 16 Pos Hujan Kabupaten Ngawi

Observasi ke-	Tahun	Bulan ke-	Hari ke-	Curah Hujan (mm)			
				Kendal	Legundi	Gentong	Paron
1	1990	3	1	31	67	10	6
2	1990	3	2	0	0	0	0
3	1990	3	3	16	5	10	4
4	1990	3	4	25	0	0	0
5	1990	3	5	2	34	10	15
6	1990	3	6	0	3	0	0
7	1990	3	7	0	1	0	0
8	1990	3	8	0	0	0	0
9	1990	3	9	2	0	0	0
10	1990	3	10	0	0	0	0
11	1990	3	11	0	0	0	0
12	1990	3	12	1	0	0	0
13	1990	3	13	0	0	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
306	1990	12	31	0	0	0	32
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
9103	2015	1	1	0	0	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
9437	2015	11	30	15	0	0	13

Observasi ke-	Tahun	Bulan ke-	Hari ke-	Curah Hujan (mm)				
				Gemarang	Kricak	Widodaren	Kedungharjo	Guyung
1	1990	3	1	0	64	9	17	25
2	1990	3	2	8	0	0	0	0
3	1990	3	3	0	25	23	25	4
4	1990	3	4	0	0	0	0	0
5	1990	3	5	0	0	0	0	0
6	1990	3	6	0	0	3	8	0
7	1990	3	7	0	0	0	0	0
8	1990	3	8	0	0	17	36	0
9	1990	3	9	0	11	6	11	0
10	1990	3	10	0	0	0	0	0
11	1990	3	11	0	0	0	0	0
12	1990	3	12	0	0	0	0	0
13	1990	3	13	0	0	0	0	0
14	1990	3	14	0	0	0	0	0
15	1990	3	15	0	0	47	27	0
16	1990	3	16	0	0	0	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
306	1990	12	31	29	0	65	23	10
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
9103	2015	1	1	6	7	30	48	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
9437	2015	11	30	44	39	0	40	0

Lampiran 9 Data Sampel *Training* 9 Pos Hujan Kabupaten Ngawi

Sample ke-	Curah Hujan Ekstrem (mm)								
	Kendal	Legundi	Gentong	Paron	Gemarang	Kricak	Widodaren	Kedungharjo	Guyung
1	89	108	120	106	89	156	63	51	168
2	63	37	0	4	15	23	23	23	14
3	50	75	130	90	58	55	50	111	63
4	71	70	125	58	82	67	138	115	86
5	78	112	75	95	54	59	69	87	72
6	0	0	0	0	0	0	2	5	0
7	51	57	95	97	86	45	110	69	68
8	155	87	125	118	76	96	75	92	82
9	96	96	65	67	56	69	87	105	57
10	124	51	90	55	36	34	61	45	26,5
11	80	70	62	81	95	98	81	69	34
12	116	126	120	122	131	87	103	121	89
13	95	60	90	59	24	70	39	71	71
14	88	54	73	48	75	50	56	55	53
15	7	6	17	11	23	27	34	26	27
16	71	153	64	81	61	63	85	67	97
17	114	90	110	72	82	54	62	59	136
18	0	0	0	0	5	0	12	22	0
19	52	42	65	80	55	55	39	32	51
20	109	108	94	81	109	98	83	121	47

Sample ke-	Curah Hujan Ekstrem (mm)								
	Kendal	**Legundi**	**Gentong**	**Paron**	**Gemarang**	**Kricak**	**Widodaren**	**Kedungharjo**	**Guyung**
21	104	126	97	47	85	96	71	53	112
22	25	54	86	37	135	81	20	21	40
23	89	97	110	40	90	138	85	115	78
24	66	95	84	59	115	95	55	82	94
25	114	44	117	150	137	125	57	77	117
26	83	46	44	31	23	12	35	69	36
27	86	62	74	85	99	61	113	42	107
28	143	76	55	127	120	85	282	153	37
29	64	37	47	86	100	78	62	71	87
30	93	12	37	56	98	41	15	31	0
31	20	42	40	41	55	45	58	67	76
32	82	118	72	96	132	116	88	97	97
33	89	145	79	49	58	82	63	65	31,3
34	36	23	63	72	70	50	96	53	14,2
35	47	70	75	115	86	97	94	80	54
36	82	67	169	92	94	90	96	150	106
37	76	92	87	60	68	80	112	150	98
38	26	35	36	45	30	6	108	180	82
39	60	201	60	65	98	84	110	65	97
40	97	81	110	111	118	115	108	108	71

Sample ke-	Curah Hujan Ekstrem (mm)								
	Kendal	**Legundi**	**Gentong**	**Paron**	**Gemarang**	**Kricak**	**Widodaren**	**Kedungharjo**	**Guyung**
41	92	107	70	93	92	98	80	240	80
42	26	67	35	51	30	18	30	30	97
43	100	131	163	110	80	90	109	77	37
44	80	141	79	41	120	62	115	65	89
45	90	87	90	45	95	80	112	125	94
46	0	61	0	0	54	97	0	109	41
47	38	136	81	31,2	149	37	85	65	77
48	60	143	81	23,5	90	125	117	90	66
49	95	85	90	45	80	49	112	47	65
50	14	0	0	0	17	0	0	0	43
51	85	73	81	78	90	98	76	82	57
52	97	89	135	133	90	98	98	79	125
53	115	39	112	105	80	68	75	79	49
54	13	16	0	3	0	27	8	15	54
55	70	60	97	97	19,4	97	64	58	98
56	113	60	100	87	57	90	70	100	95
57	97	56	100	161	94	20	77	80	39
58	38	0	19	8	0	40	65	25	29
59	18	0	0	20	19	0	43	25	0
60	105	76	87	126	100	92	95	98	124
61	82	135	50	74	60	95	84	53	121
62	54	65	18	34	95	49	58	40	98

Sample ke-	Curah Hujan Ekstrem (mm)								
	Kendal	Legundi	Gentong	Paron	Gemarang	Kricak	Widodaren	Kedungharjo	Guyung
63	28	65	41	101	54	62	57	20	99
64	50	100	45	129	95	70	57	80	116
65	69	80	57	58	25	31,6	70	70	101
66	7	0	12	14	0	6,1	0	0	0
67	32	0	45	18	60	20,7	56	50	47
68	66	80	72	79	75	50	91	80	60
69	123	63	144	65	96	60	112	110	122
70	24	40	60	32	0	0	47	0	80
71	133	50	54	88	36	76	65	70	62
72	87	70	96	115	160	150	85	75	101
73	103	65	77	67	45	32	90	80	107
74	0	0	7	48	0	25	110	80	63
75	84	80	89	105	45	28	92	55	122
76	97	80	88	71	40	73	90	81	105
77	145	85	125	81	85	98	140	75	91
78	80	85	33	12	20	21	50	0	10
79	100	70	64	61	40	42	80	58	120
80	158	85	79	96	78	70	96	61	118
81	117	86	66	130	80	70	97	90	118
82	57	56	36	60	40	30	46	34	18
83	98	82	39	65	87	68	80	138	120

Lampiran 10 Data Sampel *Testing* 9 Pos Hujan Kabupaten Ngawi

Sample ke-	Curah Hujan Ekstrem (mm)								
	Kendal	Legundi	Gentong	Paron	Gemarang	Kricak	Widodaren	Kedungharjo	Guyung
1	156	84	75	96	75	35	90	87	83
2	90	74	59	135	70	79	50	64	78
3	63	0	46	43	82	30	56	65	99
4	94	64	46	88	80	83	94	68	124
5	119	72	53	82	70	58	90	126	114
6	66	51	79	105	73	48	45	41	123
7	30	4	76	30	25	22	53	42	26
8	67	23	68	39	40	46	65	55	57
9	93	69	81	125	80	58	88	90	96
10	102	62	82	98	95	82	90	96	108
11	78	60	100	27	45	48	35	23	128
12	66	51	84	48	41	58	85	56	84
13	34	76	59	49	75	61	68	93	115
14	45	66	68	72	57	67	58	44	130
15	86	20	61	28	29	28	82	47	40
16	35	50	77	97	27	56	78	72	45
17	70	78	100	85	60	68	98	60	120
18	66	85	60	190	59	89	85	94	55
19	64	23	0	0	13	8	80	0	7
20	60	25	50	47	36	49	60	22	25

Lampiran 11 Data Transformasi dari Data Sampel *Training*

Sample ke-	Transformasi Nilai Curah Hujan Ekstrem								
	Kendal	Legundi	Gentong	Paron	Gemarang	Kricak	Widodaren	Kedungharjo	Guyung
1	2,241	4,913	8,477	5,587	2,761	188,547	1,160	0,918	700,493
2	1,043	0,643	0,291	0,317	0,401	0,488	0,388	0,433	0,358
3	0,758	1,737	13,763	2,999	1,074	1,149	0,804	5,446	1,091
4	1,294	1,508	10,718	1,104	2,178	1,664	12,148	6,190	2,173
5	1,583	5,655	1,654	3,603	0,968	1,295	1,379	2,592	1,405
6	0,284	0,284	0,291	0,294	0,301	0,289	0,226	0,273	0,276
7	0,776	1,061	3,139	3,888	2,489	0,863	4,801	1,526	1,253
8	129,458	2,478	10,718	9,641	1,799	4,780	1,642	3,015	1,907
9	2,862	3,288	1,246	1,425	1,019	1,776	2,349	4,505	0,932
10	9,971	0,909	2,647	1,018	0,626	0,643	1,095	0,778	0,460
11	1,681	1,508	1,149	2,202	3,432	5,195	1,962	1,526	0,540
12	6,597	9,543	8,477	11,790	19,119	3,349	3,846	7,520	2,405
13	2,760	1,148	2,647	1,135	0,482	1,835	0,594	1,617	1,365
14	2,168	0,981	1,560	0,847	1,745	0,993	0,951	1,026	0,842
15	0,320	0,321	0,404	0,365	0,472	0,538	0,519	0,468	0,465
16	1,294	31,012	1,212	2,202	1,165	1,465	2,211	1,441	3,204
17	5,999	2,719	5,519	1,655	2,178	1,115	1,127	1,148	22,002
18	0,284	0,284	0,291	0,294	0,331	0,289	0,291	0,421	0,276
19	0,795	0,726	1,246	2,131	0,993	1,149	0,594	0,548	0,801
20	4,792	4,913	3,032	2,202	6,065	5,195	2,082	7,520	0,727

Sample ke-	Transformasi Nilai Curah Hujan Ekstrem								
	Kendal	Legundi	Gentong	Paron	Gemarang	Kricak	Widodaren	Kedungharjo	Guyung
21	3,888	9,543	3,369	0,826	2,406	4,780	1,461	0,970	5,930
22	0,446	0,981	2,321	0,646	24,747	2,682	0,359	0,411	0,617
23	2,241	3,396	5,519	0,695	2,861	43,716	2,211	6,190	1,682
24	1,129	3,184	2,178	1,135	7,998	4,588	0,924	2,232	2,869
25	5,999	0,762	7,414	73,500	28,373	19,379	0,978	1,926	7,490
26	1,845	0,801	0,730	0,562	0,472	0,376	0,533	1,526	0,564
27	2,030	1,211	1,606	2,518	4,000	1,377	5,287	0,717	4,767
28	36,575	1,787	0,957	15,409	10,260	3,106	6425,641	22,402	0,577
29	1,071	0,643	0,785	2,606	4,161	2,410	1,127	1,617	2,247
30	2,571	0,365	0,621	1,046	3,848	0,773	0,315	0,534	0,276
31	0,405	0,726	0,665	9,712	0,993	0,863	1,006	1,441	1,583
32	1,788	7,033	1,516	3,742	20,356	11,933	2,421	3,513	3,204
33	2,241	21,280	1,865	0,869	1,074	2,781	1,160	1,361	0,509
34	0,557	0,465	1,180	1,655	1,502	0,993	3,092	0,970	0,360
35	0,708	1,508	1,654	8,348	2,489	4,982	2,907	2,104	0,863
36	1,788	1,387	247,365	3,224	3,307	3,758	3,092	20,140	4,572
37	1,492	2,895	2,398	1,167	1,417	2,588	5,119	20,140	3,327
38	0,454	0,613	0,607	0,786	0,548	0,329	4,505	60,920	1,907
39	0,966	661,905	1,089	1,344	3,848	2,993	4,801	1,361	3,204
40	2,968	2,069	5,519	6,946	9,267	11,343	4,505	4,951	1,365

Sample ke-	Transformasi Nilai Curah Hujan Ekstrem								
	Kendal	Legundi	Gentong	Paron	Gemarang	Kricak	Widodaren	Kedungharjo	Guyung
41	2.482	4,745	1,432	3,345	3,073	5,195	1,904	783,698	1,790
42	0,454	1,387	0,593	0,916	0,548	0,433	0,467	0,520	3,204
43	3,323	11,654	133,698	6,643	2,041	3,758	4,650	1,926	0,577
44	1,681	17,793	1,865	0,712	10,260	1,420	5,639	1,361	2,405
45	2,318	2,478	2,647	0,786	3,432	2,588	5,119	8,576	2,869
46	0,284	1,179	0,291	0,294	0,968	4,982	0,215	5,110	0,632
47	0,581	14,341	1,982	0,564	74,213	0,695	2,211	1,361	1,631
48	0,966	19,444	1,982	0,475	2,861	19,379	6,018	2,837	1,185
49	2,760	2,332	2,647	0,786	2,041	0,965	5,119	0,822	1,153
50	0,362	0,284	0,291	0,294	0,418	0,289	0,215	0,241	0,662
51	1,966	1,641	1,982	1,997	2,861	5,195	1,691	2,232	0,932
52	2,968	2,636	17,991	21,841	2,861	5,195	3,290	2,043	11,306
53	6,289	0,675	5,990	5,357	2,041	1,719	1,642	2,043	0,763
54	0,356	0,398	0,291	0,311	0,301	0,538	0,263	0,352	0,863
55	1,258	1,148	3,369	3,888	0,438	4,982	1,194	1,116	3,327
56	5,728	1,148	3,754	2,698	1,046	3,758	1,419	3,854	2,976
57	2,968	1,033	3,754	213,558	3,307	0,454	1,742	2,104	0,604
58	0,581	0,284	0,421	0,344	0,301	0,753	1,228	0,456	0,485
59	0,390	0,284	0,291	0,441	0,435	0,289	0,663	0,456	0,276
60	4,049	1,787	2,398	14,583	4,161	4,066	2,998	3,623	10,708
61	1,788	13,749	0,844	1,760	1,134	4,588	2,146	0,970	9,143
62	0,834	1,313	0,413	0,602	3,432	0,965	1,006	0,679	3,327

Sample ke-	Transformasi Nilai Curah Hujan Ekstrem								
	Kendal	Legundi	Gentong	Paron	Gemarang	Kricak	Widodaren	Kedungharjo	Guyung
63	0,473	1,313	0,681	4,548	0,968	1,420	0,978	0,400	3,456
64	0,758	3,747	0,748	17,247	3,432	1,835	0,978	2,104	7,138
65	1,224	2,008	1,007	1,104	0,492	0,604	1,419	1,571	3,735
66	0,320	0,284	0,366	0,388	0,301	0,330	0,215	0,241	0,276
67	0,512	0,284	0,748	0,422	1,134	0,462	0,951	0,893	0,727
68	1,129	2,008	1,516	2,062	1,745	0,993	2,652	2,104	1,007
69	9,439	1,244	30,757	1,344	3,564	1,335	5,119	5,275	9,630
70	0,437	0,691	1,089	0,575	0,301	0,289	0,740	0,241	1,790
71	17,212	0,886	0,933	2,794	0,626	2,248	1,228	1,571	1,062
72	2,097	1,508	3,252	8,348	249,311	109,421	2,211	1,816	3,735
73	3,735	1,313	1,755	1,425	0,773	0,610	2,573	2,104	4,767
74	0,284	0,284	0,332	0,847	0,301	0,512	4,801	2,104	1,091
75	1,904	2,008	2,560	5,357	0,773	0,552	2,734	1,026	9,630
76	2,968	2,008	2,477	1,605	0,687	2,029	2,573	2,167	4,386
77	43,646	2,332	10,718	2,202	2,406	5,195	13,016	1,816	2,578
78	1,681	2,332	0,568	0,373	0,444	0,465	0,804	0,241	0,332
79	3,323	1,508	1,212	1,200	0,687	0,794	1,904	1,116	8,689
80	197,521	2,332	1,865	3,742	1,915	1,835	3,092	1,214	7,864
81	6,925	2,404	1,280	18,270	2,041	1,835	3,189	2,837	7,864
82	0,897	1,033	0,607	1,167	0,687	0,580	0,720	0,578	0,387
83	3,081	2,131	0,650	1,344	2,576	1,719	1,904	13,273	8,689

Lampiran 12 Hasil Kuadrat Selisih Nilai Prediksi dan Aktual GEV

Periode	Kuadrat Selisih								
	Kendal	Legundi	Gentong	Paron	Gemarang	Kricak	Widodaren	Kedungharjo	Guyung
1 tahun	3051,524	187,118	514,559	1672,839	184,032	27,163	37,100	99,632	604,364
2 tahun	1409,559	1286,226	1464,409	489,343	1020,070	560,839	909,819	21,376	44,293
3 tahun	870,793	2203,461	705,790	174,574	757,459	1066,000	1848,321	70,137	5,383
4 tahun	603,208	2933,652	1052,801	57,462	1082,130	1474,150	2671,794	295,470	0,779
5 tahun	444,522	3416,363	1343,794	3428,056	1347,926	1335,764	2942,075	567,619	20,923
Jumlah =	46304,671								

Lampiran 13 Hasil Kuadrat Selisih Nilai Prediksi dan Aktual Frechet

Periode	**Kuadrat Selisih**								
	Kendal	**Legundi**	**Gentong**	**Paron**	**Gemarang**	**Kricak**	**Widodaren**	**Kedungharjo**	**Guyung**
1 tahun	23280,502	6484,788	5119,173	17296,391	6161,386	6319,210	8188,222	6970,465	14517,300
2 tahun	22068,860	5846,099	5111,915	16240,057	5534,865	5684,365	7462,540	14015,473	13545,247
3 tahun	20884,527	5237,947	7819,734	15212,620	6935,723	5080,573	6767,581	13056,888	13516,960
4 tahun	19728,708	4661,149	7114,248	14215,182	6264,767	4508,655	6104,231	12128,938	13024,701
5 tahun	18602,628	4245,805	6440,236	28935,265	5626,164	4761,425	6081,197	11232,687	12095,018
Jumlah =	470130,415								

Lampiran 14 Hasil Selisih Nilai Prediksi dan Aktual GEV dibagi Nilai Aktual

Periode	Kuadrat Selisih dibagi Nilai Aktual								
	Kendal	**Legundi**	**Gentong**	**Paron**	**Gemarang**	**Kricak**	**Widodaren**	**Kedungharjo**	**Guyung**
1 tahun	0,35411	0,16285	0,30245	0,30297	0,16544	0,06279	0,06480	0,11473	0,19826
2 tahun	0,24067	0,42695	0,48440	0,16386	0,38949	0,28533	0,32089	0,03669	0,05367
3 tahun	0,18916	0,55882	0,26567	0,09787	0,28970	0,39337	0,45736	0,06647	0,01813
4 tahun	0,15744	0,64480	0,32447	0,05615	0,34627	0,46259	0,54989	0,13642	0,00679
5 tahun	0,13515	0,68764	0,36658	0,30816	0,38646	0,41065	0,55348	0,18909	0,03519
Jumlah =	12,224								

Lampiran 15 *Syntax Software* R

```
#Package yang harus diinstall
1.extrem
2.nsrfa
3.gstat
4.spatialextrem

#Input Koordinat Lokasi
loc<-read.table("D:/Document/Tesis/BISMILLAH/Data/DATA
NOTEPAD/KOORDINAT.txt", header=T)
loc<-as.matrix(loc)
colnames(loc)<-c("lon", "lat")

#Input Data Ekstrem
b1<-as.matrix(read.table("D:/Document/Tesis/BISMILLAH/Data/
DATA NOTEPAD/KENDAL.txt", header=T))
b2<-as.matrix(read.table("D:/Document/Tesis/BISMILLAH/Data/
DATA NOTEPAD/LEGUNDI.txt", header=T))
b3<-as.matrix(read.table("D:/Document/Tesis/BISMILLAH/Data/
DATA NOTEPAD/GENTONG.txt", header=T))
b4<-as.matrix(read.table("D:/Document/Tesis/BISMILLAH/Data/
DATA NOTEPAD/PARON.txt", header=T))
b5<-as.matrix(read.table("D:/Document/Tesis/BISMILLAH/Data/
DATA NOTEPAD/GEMARANG.txt", header=T))
b6<-as.matrix(read.table("D:/Document/Tesis/BISMILLAH/Data/
DATA NOTEPAD/KRICAK.txt", header=T))
b7<-as.matrix(read.table("D:/Document/Tesis/BISMILLAH/Data/
DATA NOTEPAD/WIDODAREN.txt", header=T))
b8<-as.matrix(read.table("D:/Document/Tesis/BISMILLAH/Data/
DATA NOTEPAD/KEDUNGHARJO.txt", header=T))
b9<-as.matrix(read.table("D:/Document/Tesis/BISMILLAH/Data/
DATA NOTEPAD/GUYUNG.txt", header=T))
```

```
B=matrix(c(b1,b2,b3,b4,b5,b6,b7,b8,b9),ncol=9)
colnames(B)=c("KENDAL","LEGUNDI","GENTONG","PARON","GEMAR
ANG","KRICAK","WIDODAREN","KEDUNGHARJO","GUYUNG")

#Uji Anderson Darling
AD1<-A2_GOFlaio(b1, dist="GEV")
AD1
AD2<-A2_GOFlaio(b2, dist="GEV")
AD2
AD3<-A2_GOFlaio(b3, dist="GEV")
AD3
AD4<-A2_GOFlaio(b4, dist="GEV")
AD4
AD5<-A2_GOFlaio(b5, dist="GEV")
AD5
AD6<-A2_GOFlaio(b6, dist="GEV")
AD6
AD7<-A2_GOFlaio(b7, dist="GEV")
AD7
AD8<-A2_GOFlaio(b8, dist="GEV")
AD8
AD9<-A2_GOFlaio(b9, dist="GEV")
AD9

#transformasi GEV ke Frechet
z1 <- gev2frech(b1, 61.34900, 39.12917, -0.33648)
z2 <- gev2frech(b2, 54.73548, 38.60414, -0.18410)
z3 <- gev2frech(b3, 56.72068, 38.80116, -0.26563)
z4 <- gev2frech(b4, 54.34053, 37.47343, -0.26780)
z5 <- gev2frech(b5, 55.27276, 38.42322, -0.29477)
z6 <- gev2frech(b6, 50.24568, 34.87386, -0.23194)
z7 <- gev2frech(b7, 57.78616, 35.37236, -0.07849)
z8 <- gev2frech(b8, 54.08141, 35.88077, -0.07897)
```

```
z9 <- gev2frech(b9, 59.71857, 37.98354, -0.30242)
Z=matrix(c(z1,z2,z3,z4,z5,z6,z7,z8,z9),ncol=9)
colnames(Z)=c("KENDAL","LEGUNDI","GENTONG","PARON","GEMARAN
G","KRICAK","WIDODAREN","KEDUNGHARJO","GUYUNG")

#extremal coefficient
fitextcoeff(Z, loc, estim = "Smith")

#grafik extremal coefficient
fmadogram(Z,loc,which="ext",col="black", estim="smith")

#TIC
loc.form<-z~lon+lat
scale.form<-z~lon+lat
shape.form<-z~1
H1<-fitmaxstab(Z, loc, loc.form, scale.form,
shape.form,cov.mod="gauss",iso=TRUE,method="BFGS")

loc.form<-z~lon
scale.form<-z~lon+lat
shape.form<-z~1
H2<-fitmaxstab(Z, loc, loc.form, scale.form,
shape.form,cov.mod="gauss",iso=TRUE,method="BFGS")

loc.form<-z~lon+lat
scale.form<-z~lon
shape.form<-z~1
H3<-fitmaxstab(Z, loc, loc.form, scale.form,
shape.form,cov.mod="gauss",iso=TRUE,method="BFGS")
```

```
loc.form<-z~lon
scale.form<-z~lon
shape.form<-z~1
H4<-fitmaxstab(Z, loc, loc.form, scale.form,
shape.form,cov.mod="gauss",iso=TRUE,method="BFGS")

loc.form<-z~lat
scale.form<-z~lon+lat
shape.form<-z~1
H5<-fitmaxstab(Z, loc, loc.form, scale.form,
shape.form,cov.mod="gauss",iso=TRUE,method="BFGS")

loc.form<-z~lon+lat
scale.form<-z~lat
shape.form<-z~1
H6<-fitmaxstab(Z, loc, loc.form, scale.form,
shape.form,cov.mod="gauss",iso=TRUE,method="BFGS")

loc.form<-z~lat
scale.form<-z~lat
shape.form<-z~1
H7<-fitmaxstab(Z, loc, loc.form, scale.form,
shape.form,cov.mod="gauss",iso=TRUE,method="BFGS")

loc.form<-z~lon
scale.form<-z~lat
shape.form<-z~1
H8<-fitmaxstab(Z, loc, loc.form, scale.form,
shape.form,cov.mod="gauss",iso=TRUE,method="BFGS")
```

```
loc.form<-Z~lat
scale.form<-Z~lon
shape.form<-Z~1
H9<-fitmaxstab(Z, loc, loc.form, scale.form,
shape.form,cov.mod="gauss",iso=TRUE,method="BFGS")

TIC(H1,H2,H3,H4,H5,H6,H7,H8,H9)

#Estimasi parameter Smith dengan Model Terbaik
loc.form<-z~lon
scale.form<-z~lon
shape.form<-z~1
Smith <-fitmaxstab(Z, loc, loc.form, scale.form,
shape.form,cov.mod="gauss",iso=TRUE,method="BFGS")
Smith

#Return Level 1-5 tahun
s1=predict(Smith, loc, ret.per=4)
s1
s2=predict(Smith, loc, ret.per=8)
s2
s3=predict(Smith, loc, ret.per=12)
s3
s4=predict(Smith, loc, ret.per=16)
s4
s5=predict(Smith, loc, ret.per=20)
s5

#Input Data Prediksi Return Level
e1<-as.matrix(read.table("D:/Document/Tesis/BISMILLAH/
Data/DATA NOTEPAD/e1.txt", header=T))
e2<-as.matrix(read.table("D:/Document/Tesis/BISMILLAH/
Data/DATA NOTEPAD/e2.txt", header=T))
```

```
e3<-as.matrix(read.table("D:/Document/Tesis/BISMILLAH/
Data/DATA NOTEPAD/e3.txt", header=T))
e4<-as.matrix(read.table("D:/Document/Tesis/BISMILLAH/
Data/DATA NOTEPAD/e4.txt", header=T))
e5<-as.matrix(read.table("D:/Document/Tesis/BISMILLAH/
Data/DATA NOTEPAD/e5.txt", header=T))
e6<-as.matrix(read.table("D:/Document/Tesis/BISMILLAH/
Data/DATA NOTEPAD/e6.txt", header=T))
e7<-as.matrix(read.table("D:/Document/Tesis/BISMILLAH/
Data/DATA NOTEPAD/e7.txt", header=T))
e8<-as.matrix(read.table("D:/Document/Tesis/BISMILLAH/
Data/DATA NOTEPAD/e8.txt", header=T))
e9<-as.matrix(read.table("D:/Document/Tesis/BISMILLAH/
Data/DATA NOTEPAD/e9.txt", header=T))

#Transformasi Frechet ke GEV Return Level
prediksi1<- frech2gev(e1, 61.34900, 39.12917, -0.33648)
prediksi1
prediksi2<- frech2gev(e2, 54.73548, 38.60414, -0.18410)
prediksi2
prediksi3<- frech2gev(e3, 56.72068, 38.80116, -0.26563)
prediksi3
prediksi4<- frech2gev(e4, 54.34053, 37.47343, -0.26780)
prediksi4
prediksi5<- frech2gev(e5, 55.27276, 38.42322, -0.29477)
prediksi5
prediksi6<- frech2gev(e6, 50.24568, 34.87386, -0.23194)
prediksi6
prediksi7<- frech2gev(e7, 57.78616, 35.37236, -0.07849)
prediksi7
prediksi8<- frech2gev(e8, 54.08141, 35.88077, -0.07897)
prediksi8
prediksi9<- frech2gev(e9, 59.71857, 37.98354, -0.30242)
prediksi9
```

Lampiran 16 *Output* R Estimasi Parameter Univariat

```
   GEV fit
-----------------------------------
Response variable: KENDAL



 Likelihood ratio test (5% level) for xi=0 does not accept
Gumbel hypothesis.
likelihood ratio statistic is  14.79782  >  3.841459  1 df
chi-square critical value.

 p-value for likelihood-ratio test is  0.0001196737



 Convergence successfull![1] "Convergence successfull!"
[1] "Maximum Likelihood Estimates:"
                       MLE Stand. Err.
MU: (identity)    61.34900     4.69858
SIGMA: (identity) 39.12917     3.34059
Xi: (identity)    -0.33648     0.06519



[1] "Negative log-likelihood: 419.128426880521"

 Parameter covariance:
           [,1]       [,2]         [,3]
[1,] 22.0766827 -1.0328140 -0.114986937
[2,] -1.0328140 11.1595673 -0.133704559
[3,] -0.1149869 -0.1337046  0.004250217
[1] "Convergence code (see help file for optim): 0"
NULL
Model name: gev.fit2
```

```
   GEV fit
-----------------------------------
Response variable: LEGUNDI



 Likelihood ratio test (5% level) for xi=0 does not accept
Gumbel hypothesis.
likelihood ratio statistic is  6.261444  >  3.841459  1 df
chi-square critical value.

 p-value for likelihood-ratio test is  0.01233936



 Convergence successfull![1] "Convergence successfull!"
[1] "Maximum Likelihood Estimates:"
                       MLE Stand. Err.
MU: (identity)    54.73548     4.62351
SIGMA: (identity) 38.60414     3.17549
Xi: (identity)    -0.18410     0.05687



[1] "Negative log-likelihood: 424.733788424043"

 Parameter covariance:
            [,1]        [,2]         [,3]
[1,] 21.37685568  1.71685316 -0.087489162
[2,]  1.71685316 10.08376550 -0.082804915
[3,] -0.08748916 -0.08280492  0.003234331
[1] "Convergence code (see help file for optim): 0"
NULL
Model name: gev.fit1
```

```
   GEV fit
-----------------------------------
Response variable: GENTONG


 Likelihood ratio test (5% level) for xi=0 does not accept
Gumbel hypothesis.
likelihood ratio statistic is  10.18033  >  3.841459  1 df
chi-square critical value.

 p-value for likelihood-ratio test is  0.001419468


 Convergence successfull![1] "Convergence successfull!"
[1] "Maximum Likelihood Estimates:"
                       MLE Stand. Err.
MU: (identity)    56.72068     4.66963
SIGMA: (identity) 38.80116     3.26344
Xi: (identity)    -0.26563     0.06392


[1] "Negative log-likelihood: 421.744223900704"

 Parameter covariance:
           [,1]       [,2]         [,3]
[1,] 21.8054763  0.4034799 -0.109388233
[2,]  0.4034799 10.6500717 -0.116301595
[3,] -0.1093882 -0.1163016  0.004086164
[1] "Convergence code (see help file for optim): 0"
NULL
Model name: gev.fit1
```

```
   GEV fit
-----------------------------------
Response variable: KRICAK



 Likelihood ratio test (5% level) for xi=0 does not accept
Gumbel hypothesis.
likelihood ratio statistic is  6.736406  >  3.841459  1 df
chi-square critical value.

 p-value for likelihood-ratio test is  0.009446476



 Convergence successfull![1] "Convergence successfull!"
[1] "Maximum Likelihood Estimates:"
                       MLE Stand. Err.
MU: (identity)    50.24568     4.24274
SIGMA: (identity) 34.87386     3.00527
Xi: (identity)    -0.23194     0.07138



[1] "Negative log-likelihood: 414.787392100303"

 Parameter covariance:
           [,1]       [,2]         [,3]
[1,] 18.0008246  1.2512573 -0.115861747
[2,]  1.2512573  9.0316249 -0.117021361
[3,] -0.1158617 -0.1170214  0.005095454
[1] "Convergence code (see help file for optim): 0"
NULL
Model name: gev.fit1
```

```
   GEV fit
-----------------------------------
Response variable: WIDODAREN



 Likelihood ratio test (5% level) for xi=0 does not reject
Gumbel hypothesis.
likelihood ratio statistic is  2.444553  <  3.841459  1 df
chi-square critical value.

 p-value for likelihood-ratio test is  0.1179335



 Convergence successfull![1] "Convergence successfull!"
[1] "Maximum Likelihood Estimates:"
                       MLE Stand. Err.
MU: (identity)    57.78616     4.18970
SIGMA: (identity) 35.37236     2.80506
Xi: (identity)    -0.07849     0.04076



[1] "Negative log-likelihood: 420.47482288364"

 Parameter covariance:
            [,1]        [,2]         [,3]
[1,] 17.55359855  2.71162525 -0.043190745
[2,]  2.71162525  7.86835958 -0.029064248
[3,] -0.04319074 -0.02906425  0.001661501
[1] "Convergence code (see help file for optim): 0"
NULL
Model name: gev.fit1
```

```
   GEV fit
-----------------------------------
Response variable: KEDUNGHARJO



 Likelihood ratio test (5% level) for xi=0 does not reject
Gumbel hypothesis.
likelihood ratio statistic is  1.353733  <  3.841459  1 df
chi-square critical value.

 p-value for likelihood-ratio test is  0.2446266



 Convergence successfull![1] "Convergence successfull!"
[1] "Maximum Likelihood Estimates:"
                      MLE Stand. Err.
MU: (identity)    54.08141     4.32392
SIGMA: (identity) 35.88077     3.00186
Xi: (identity)    -0.07897     0.06025



[1] "Negative log-likelihood: 423.606423390632"

 Parameter covariance:
            [,1]        [,2]         [,3]
[1,] 18.69631645  3.34771166 -0.081515114
[2,]  3.34771166  9.01118823 -0.056287631
[3,] -0.08151511 -0.05628763  0.003630628
[1] "Convergence code (see help file for optim): 0"
NULL
Model name: gev.fit1
```

```
   GEV fit
-----------------------------------
Response variable: GUYUNG



 Likelihood ratio test (5% level) for xi=0 does not accept
Gumbel hypothesis.
likelihood ratio statistic is  13.48672  >  3.841459  1 df
chi-square critical value.

 p-value for likelihood-ratio test is  0.0002402577



 Convergence successfull![1] "Convergence successfull!"
[1] "Maximum Likelihood Estimates:"
                       MLE Stand. Err.
MU: (identity)    59.71857     4.51147
SIGMA: (identity) 37.98354     3.17361
Xi: (identity)    -0.30242     0.05366



[1] "Negative log-likelihood: 417.943422393521"

 Parameter covariance:
            [,1]       [,2]         [,3]
[1,] 20.35337055 -0.6400562 -0.083534912
[2,] -0.64005623 10.0718182 -0.104640977
[3,] -0.08353491 -0.1046410  0.002879707
[1] "Convergence code (see help file for optim): 0"
NULL
Model name: gev.fit1
```

Lampiran 17 *Output* R Estimasi Parameter Model Smith

```
        Estimator: MPLE
            Model: Smith
         Weighted: FALSE
   Pair. Deviance: 26109.25
              TIC: 26237.62
Covariance Family: Gaussian

Estimates
  Marginal Parameters:
    Location Parameters:
locCoeff1  locCoeff2
    2.794      0.242
       Scale Parameters:
scaleCoeff1  scaleCoeff2
     1.8196       0.1106
       Shape Parameters:
shapeCoeff1
      1.012
  Dependence Parameters:
      cov
0.0006821

Standard Error Type: score

Standard Errors
cov locCoeff1 locCoeff2 scaleCoeff1 scaleCoeff2 shapeCoeff1
5.988e-05 4.092e+00 5.501e-01 6.813e+00 9.171e-01 4.767e-02
```

```
Asymptotic Variance Covariance
            cov       locCoef1 locCoef2 scaleCoef1 scaleCoef2
shapeCoef1
cov         3.585e-09 4.091e-05 5.441e-06 1.119e-04 1.475e-05
2.786e-06
locCoeff1   4.091e-05 1.675e+01 2.251e+00 2.043e+01 2.741e+00
2.064e-02
locCoeff2   5.441e-06 2.251e+00 3.026e-01 2.745e+00 3.684e-01
2.738e-03
scaleCoeff1 1.119e-04 2.043e+01 2.745e+00 4.642e+01 6.248e+00
7.870e-02
scaleCoeff2 1.475e-05 2.741e+00 3.684e-01 6.248e+00 8.411e-01
1.038e-02
shapeCoeff1 2.786e-06 2.064e-02 2.738e-03 7.870e-02 1.038e-02
2.273e-03

Optimization Information
  Convergence: Stayed at start. val.
  Function Evaluations: 54
  Gradient Evaluations: 1
```

Lampiran 18 Turunan Kedua Distribusi *Generalized Extreme Value*

$$\frac{\partial^2 \ell(\mu,\sigma,\xi)}{\partial\mu\partial\mu} = -\left(1+\frac{1}{\xi}\right)\left(\sum_{i=1}^{n}\left[-\frac{\left(\xi\frac{1}{\sigma}\right)^2}{\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^2}\right]\right) - \left(\sum_{i=1}^{n}\left[\frac{\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^{-\frac{1}{\xi}}\left(\xi\frac{1}{\sigma}\right)^2}{\xi^2\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^2} + \frac{\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^{-\frac{1}{\xi}}\left(\xi\frac{1}{\sigma}\right)^2}{\xi\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^2}\right]\right)$$

$$\frac{\partial^2 \ell(\mu,\sigma,\xi)}{\partial\mu\partial\sigma} = -\left(1+\frac{1}{\xi}\right)\left(\sum_{i=1}^{n}\left[-\frac{\left(\xi\frac{1}{\sigma^2}\right)}{\left(1+\xi\frac{x_i-\mu}{\sigma}\right)} - \frac{\xi\frac{1}{\sigma}\left(\xi\frac{x_i-\mu}{\sigma^2}\right)}{\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^2}\right]\right) - \left(\sum_{i=1}^{n}\left[\frac{\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^{-\frac{1}{\xi}}\left(\xi\frac{x_i-\mu}{\sigma^2}\right)\left(\xi\frac{1}{\sigma}\right)}{\xi^2\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^2} - \frac{\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^{-\frac{1}{\xi}}\left(\xi\frac{1}{\sigma^2}\right)}{\xi\left(1+\xi\frac{x_i-\mu}{\sigma}\right)} + \right.\right.$$

$$\left.\left.\frac{\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^{-\frac{1}{\xi}}\left(\xi\frac{x_i-\mu}{\sigma^2}\right)\left(\xi\frac{1}{\sigma}\right)}{\xi\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^2}\right]\right)$$

$$\frac{\partial^2 \ell(\mu,\sigma,\xi)}{\partial\mu\partial\xi}=\frac{1}{\xi^2}\left(\sum_{i=1}^{n}-\frac{\xi\frac{1}{\sigma}}{\left(1+\xi\frac{x_i-\mu}{\sigma}\right)}\right)-\left(1+\frac{1}{\xi}\right)\left(\sum_{i=1}^{n}\left[-\frac{1}{\sigma\left(1+\xi\frac{x_i-\mu}{\sigma}\right)}+\frac{\xi\frac{1}{\sigma}\left(x_i-\mu\right)}{\sigma\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^2}\right]\right)-\sum_{i=1}^{n}\left(\frac{\left[1+\xi\frac{x_i-\mu}{\sigma}\right]^{-\frac{1}{\xi}}\left[\frac{\ln\left(1+\xi\frac{x_i-\mu}{\sigma}\right)}{\xi^2}-\frac{x_i-\mu}{\xi\sigma\left(1+\xi\frac{x_i-\mu}{\sigma}\right)}\right]\xi\frac{1}{\sigma}}{\xi\left[1+\xi\frac{x_i-\mu}{\sigma}\right]}-\right.$$

$$\left.\frac{\left[1+\xi\frac{x_i-\mu}{\sigma}\right]^{-\frac{1}{\xi}}\xi\frac{1}{\sigma}}{\xi^2\left[1+\xi\frac{x_i-\mu}{\sigma}\right]}+\frac{\left[1+\xi\frac{x_i-\mu}{\sigma}\right]^{-\frac{1}{\xi}}}{\xi\sigma\left[1+\xi\frac{x_i-\mu}{\sigma}\right]}-\frac{\left[1+\xi\frac{x_i-\mu}{\sigma}\right]^{-\frac{1}{\xi}}\xi\frac{1}{\sigma}\left[x_i-\mu\right]}{\xi\sigma\left[1+\xi\frac{x_i-\mu}{\sigma}\right]^2}\right)$$

$$\frac{\partial^2 \ell(\mu,\sigma,\xi)}{\partial\sigma\partial\sigma}=-\left(1+\frac{1}{\xi}\right)\left(\sum_{i=1}^{n}\left[\frac{2\xi\frac{x_i-\mu}{\sigma^3}}{\left(1+\xi\frac{x_i-\mu}{\sigma}\right)}-\frac{\left(\xi\frac{x_i-\mu}{\sigma^2}\right)^2}{\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^2}\right]\right)-\left(\sum_{i=1}^{n}\left[\frac{\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^{-\frac{1}{\xi}}\left(\xi\frac{x_i-\mu}{\sigma^2}\right)^2}{\xi^2\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^2}-\frac{2\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^{-\frac{1}{\xi}}\left(\xi\frac{x_i-\mu}{\sigma^3}\right)}{\xi\left(1+\xi\frac{x_i-\mu}{\sigma}\right)}+\right.\right.$$

$$\left.\left.\frac{\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^{-\frac{1}{\xi}}\left(\xi\frac{x_i-\mu}{\sigma^2}\right)^2}{\xi\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^2}\right]\right)$$

$$\frac{\partial^2\ell(\mu,\sigma,\xi)}{\partial\sigma\partial\xi}=\frac{1}{\xi^2}\left(\sum_{i=1}^{n}-\frac{\xi\frac{x_i-\mu}{\sigma}}{\left(1+\xi\frac{x_i-\mu}{\sigma}\right)}\right)-\left(1+\frac{1}{\xi}\right)\left(\sum_{i=1}^{n}\left[-\frac{(x_i-\mu)}{\sigma^2\left(1+\xi\frac{x_i-\mu}{\sigma}\right)}+\frac{\xi\frac{x_i-\mu}{\sigma}(x_i-\mu)}{\sigma\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^2}\right]\right)-\left(\sum_{i=1}^{n}\left(\frac{\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^{-\frac{1}{\xi}}\left(\frac{\ln\left(1+\xi\frac{x_i-\mu}{\sigma}\right)}{\xi^2}-\frac{(x_i-\mu)}{\xi\sigma\left(1+\xi\frac{x_i-\mu}{\sigma}\right)}\right)\left(\xi\frac{x_i-\mu}{\sigma}\right)}{\xi\left(1+\xi\frac{x_i-\mu}{\sigma}\right)}-\right.\right.$$

$$\left.\left.\frac{\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^{-\frac{1}{\xi}}\left(\xi\frac{x_i-\mu}{\sigma}\right)}{\xi^2\left(1+\xi\frac{x_i-\mu}{\sigma}\right)}+\frac{\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^{-\frac{1}{\xi}}(x_i-\mu)}{\sigma^2\xi\left(1+\xi\frac{x_i-\mu}{\sigma}\right)}-\frac{\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^{-\frac{1}{\xi}}\left(\xi\frac{x_i-\mu}{\sigma}\right)(x_i-\mu)}{\sigma\xi\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^2}\right)\right)$$

$$\frac{\partial^2 \ell(\mu,\sigma,\xi)}{\partial\xi\partial\xi} = -2\frac{1}{\xi^3}\sum_{i=1}^{n}\ln\left(1+\xi\frac{x_i-\mu}{\sigma}\right)+2\frac{1}{\xi^2}\sum_{i=1}^{n}\left(\frac{x_i-\mu}{\sigma\left(1+\xi\frac{x_i-\mu}{\sigma}\right)}\right)-\left(1+\frac{1}{\xi}\right)\sum_{i=1}^{n}\left(-\frac{\left(x_i-\mu\right)^2}{\sigma^2\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^2}\right)-\sum_{i=1}^{n}\left(\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^{-\frac{1}{\xi}}\times\right.$$

$$\left.\left(\frac{\ln\left(1+\xi\frac{x_i-\mu}{\sigma}\right)}{\xi^2}-\frac{x_i-\mu}{\sigma\xi\left(1+\xi\frac{x_i-\mu}{\sigma}\right)}\right)^2+\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^{-\frac{1}{\xi}}\left(-\frac{2\ln\left(1+\xi\frac{x_i-\mu}{\sigma}\right)}{\xi^2}+\frac{2\left(x_i-\mu\right)}{\sigma\xi^2\left(1+\xi\frac{x_i-\mu}{\sigma}\right)}+\frac{\left(x_i-\mu\right)^2}{\sigma^2\xi\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^2}\right)\right)$$

Lampiran 19 Turunan Kedua Fungsi *Ln Likelihood* Model Smith (1)

$$\frac{\partial^2 \ell(\boldsymbol{\beta})}{\partial\beta_\mu\beta_\mu}=\sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}\left(-\frac{\Phi(w)\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)^2\right)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\right)^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^2}-\frac{\Phi(w)\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)^2\right)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^2}-\right.$$

$$\left.\frac{\Phi(v)\left(d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)^2\right)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}\left(d_k\beta_\xi\right)^2\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^2}-\frac{\Phi(v)\left(d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)^2\right)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}\left(d_k\beta_\xi\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^2}\right)\times$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}\ln\left(\left(\left(\frac{\Phi(w)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}+\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(v)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}}\right)\times\right.\right.$$

$$\left(\frac{\Phi(v)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}}+\frac{\varphi(v)}{a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}\right)+$$

$$\frac{v\varphi(w)}{a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}-\frac{w\varphi(v)}{a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}\Bigg)+2\Bigg(\Bigg(-\frac{\Phi(w)\left(d_j\beta_\xi\frac{x_{ji}}{d_j\beta_\sigma}\right)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}-$$

$$\frac{\Phi(v)\left(d_k\beta_\xi\frac{d_k}{d_k\beta_\sigma}\right)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}d_k\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\Bigg)\Bigg)\Bigg(\Bigg(\Bigg(\Bigg(\frac{2\Phi(w)\left(d_j\beta_\xi\frac{d_j}{d_j\beta_\sigma}\right)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}+$$

$$
\frac{2\varphi(w)\left[a(h_{j,k})\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}} d_j\beta_\xi\frac{d_j}{d_j\beta_\sigma}}{d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\right]}{\left\{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right\}^2}+\left[\varphi(v)\left(-a(h_{j,k})\left[\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}} d_j\beta_\xi\frac{d_j}{d_j\beta_\sigma}}{d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right]\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}-\right.\right.
$$

$$
\left.\left.a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}} d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)}{d_k\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right]\right)\right]\Bigg/\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}\right)^2\times
$$

$$
\left(\frac{\Phi(v)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}}+\frac{\varphi(v)}{a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}\right)+
$$

$$\left(\frac{\Phi(w)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}} + \frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}} - \frac{\varphi(v)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}} \right) \times$$

$$\left(\frac{2\Phi(v)\left(d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}} d_k\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]} + \frac{2\varphi(v)a(h_{j,k})}{\left(a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^2}\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}} d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]} + \right.$$

$$\left(\varphi(w)\left(-a(h_{j,k})\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}} d_j\beta_\xi\frac{d_j}{d_j\beta_\sigma}}{d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]} \right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}} - a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}} d_k\beta_\xi\frac{d_k}{d_k\beta_\sigma}}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]} \right)\right)\right) /$$

$$\Bigg(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\Bigg)^2\Bigg)-\Bigg(v\varphi(w)\Bigg(-2\Bigg(a(h_{j,k})^2\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)}{d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\Bigg)-$$

$$a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}d_k\beta_\xi\frac{d_k}{d_k\beta_\sigma}}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\Bigg)\Bigg)\Bigg)\Bigg/\Bigg(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\Bigg)^2+$$

$$\Bigg(w\varphi(v)\Bigg(-(a(h_{j,k})^2)\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)}{d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}-2\Bigg((a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\times$$

$$
\Bigg(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}} d_k\beta_\xi\,\frac{d_k}{d_k\beta_\sigma}}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\Bigg)\Bigg)\Bigg)\Bigg) \Bigg/ \Bigg(\Bigg(\Bigg(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\Bigg)^2\Bigg)/
$$

$$
\Bigg(\Bigg(\frac{\Phi(w)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}+\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(v)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}}\Bigg)\times
$$

$$
\Bigg(\frac{\Phi(v)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}}+\frac{\varphi(v)}{a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}\Bigg)+
$$

$$\frac{v\varphi(w)}{a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}-\frac{w\varphi(v)}{a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}\Bigg)+$$

$$\left(-\frac{\Phi(w)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}-\frac{\Phi(v)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}}\right)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg(\Bigg(\Bigg(\Bigg(\frac{4\Phi(w)\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)^2}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\right)^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^2}+$$

$$
\frac{2\Phi(w)\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)^2}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^2}+\frac{8\varphi(w)\left(a(h_{j,k})\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\right)^2}{\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^3}+
$$

$$
\frac{1}{\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^2}\left(2\varphi(w)\left(a(h_{j,k})\left(-\frac{2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)^2}{\left(d_j\beta_\xi\right)^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^2}+\right.\right.\right.
$$

$$\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)^2}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^2}\Bigg)\Bigg)\Bigg)\Bigg)-\Bigg(2\varphi(v)\Bigg(-a(h_{j,k})\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^2}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}-$$

$$a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)}{d_k\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\Bigg)\Bigg)^2\Bigg)\Bigg/\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}\right)^3+$$

$$\Bigg(\varphi(v)\Bigg(-a(h_{j,k})\left(-\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)^2}{\left(d_j\beta_\xi\right)^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^2}+\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)^2}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^2}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}+$$

$$2\Bigg(a(h_{j,k})\Bigg(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\Bigg)\Bigg(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)}{d_k\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\Bigg)\Bigg)-a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\times$$

$$\Bigg(-\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)^2}{\left(d_k\beta_\xi\right)^2\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^2}+\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)^2}{\left(d_k\beta_\xi\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^2}\Bigg)\Bigg)\Bigg)\Bigg/\Bigg(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}\Bigg)^2\Bigg)\times$$

$$\Bigg(\frac{\Phi(v)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}}+\frac{\varphi(v)}{a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}\Bigg)+2\Bigg(\Bigg(\frac{2\Phi(w)\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}+$$

$$
\frac{2\varphi(w)\Bigg(a(h_{j,k})\Bigg(\dfrac{\left[1+d_j\beta_\xi\left(\dfrac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\dfrac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\dfrac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\Bigg)\Bigg)}{\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\dfrac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^2}+\Bigg(\varphi(v)\Bigg(-a(h_{j,k})\Bigg(\frac{\left[1+d_j\beta_\xi\left(\dfrac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\dfrac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\dfrac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\Bigg)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}-
$$

$$
a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\Bigg(\frac{\left[1+d_k\beta_\xi\left(\dfrac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}d_k\beta_\xi\left(\dfrac{d_k}{d_k\beta_\sigma}\right)}{d_k\beta_\xi\left[1+d_k\beta_\xi\left(\dfrac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\Bigg)\Bigg)\Bigg)\Bigg)/\Bigg(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}\Bigg)^2\Bigg)\times
$$

$$
\Bigg(\frac{2\Phi(v)\left(d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}d_k\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}+\frac{2\varphi(v)\left(a(h_{j,k})\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\right)}{\left(a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^2}+
$$

$$
\Bigg(\varphi(w)\Bigg(-a(h_{j,k})\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}-a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\times
$$

$$
\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\Bigg)\Bigg)\Bigg/\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\right)^2\Bigg)\Bigg)+
$$

$$\left(\frac{\Phi(w)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}+\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(v)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}}\right)\times$$

$$\left(\frac{4\Phi(v)\left(d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)^2}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}\left(d_k\beta_\xi\right)^2\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^2}+\frac{2\Phi(v)\left(d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)^2}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}\left(d_k\beta_\xi\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^2}+\right.$$

$$\frac{8\varphi(v)\left(a(h_{j,k})\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\right)^2}{\left(a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^3}+\frac{1}{\left(a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^2}\left(2\varphi(v)\left(a(h_{j,k})\left(-\frac{2\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)^2}{\left(d_j\beta_\xi\right)^2\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^2}+\right.\right.\right.$$

$$\left.\left.\left.\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)^2}{\left(d_j\beta_\xi\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^2}\right)\right)\right)-\left(2\varphi(w)\left(-a(h_{j,k})\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}d_j\beta_\xi\frac{d_j}{d_j\beta_\sigma}}{d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}-\right.\right.$$

$$\left.\left.a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}d_k\beta_\xi\frac{d_k}{d_k\beta_\sigma}}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\right)^2\right)/\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\right)^3+$$

$$\left(\varphi(w)\left(-a(h_{j,k})\left(-\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)^2}{\left(d_j\beta_\xi\right)^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^2}+\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)^2}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^2}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}+\right.\right.$$

$$
2\Bigg(a(h_{j,k})\Bigg(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\Bigg)\Bigg(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}d_k\beta_\xi\frac{d_k}{d_k\beta_\sigma}}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\Bigg)\Bigg)-a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\times
$$

$$
\Bigg(-\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)^2}{\left(d_j\beta_\xi\right)^2\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^2}+\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)^2}{\left(d_j\beta_\xi\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^2}\Bigg)\Bigg)\Bigg)\Bigg)/\Bigg(\Bigg(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\Bigg)^2\Bigg)+
$$

$$
\Bigg(2v\varphi w\Bigg(-2a(h_{j,k})^2\Bigg(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\Bigg)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}-a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\times
$$

$$\left.\left.\left.\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\right)^2\right)\right/\left(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\right)^3-$$

$$\left(v\varphi(w)\left(-2\left(a(h_{j,k})^2\left(-\frac{2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)^2}{\left(d_j\beta_\xi\right)^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^2}+\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)^2}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^2}\right)\times\right.\right.\right.$$

$$\left.\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\right)+4\left(a(h_{j,k})^2\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}d_k\beta_\xi\frac{d_k}{d_k\beta_\sigma}}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\right)-$$

$$a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\Bigg(-\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)^2}{\left(d_j\beta_\xi\right)^2\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^2}+\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)^2}{\left(d_j\beta_\xi\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^2}\Bigg)\Bigg)\Bigg)\Bigg)/$$

$$\Bigg(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\Bigg)^2-\Bigg(2v\varphi(w)\Bigg(-a(h_{j,k})^2\Bigg(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\Bigg)\times$$

$$\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}-2\Bigg(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\Bigg(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\Bigg)\Bigg)\Bigg)\Bigg)^2\Bigg)/$$

$$\Bigg(a(h_{j,k})^2 \Big[1+d_j\beta_\xi\Big(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\Big)\Big]^{\frac{1}{d_j\beta_\xi}} \Big[1+d_k\beta_\xi\Big(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\Big)\Big]^{\frac{2}{d_j\beta_\xi}} \Bigg)^3 + \big(w\varphi(v)\Bigg(-a(h_{j,k})^2 \Bigg(-\Bigg(\frac{\Big[1+d_j\beta_\xi\Big(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\Big)\Big]^{\frac{1}{d_j\beta_\xi}} \Big(d_j\beta_\xi\Big(\frac{d_j}{d_j\beta_\sigma}\Big)\Big)^2}{\big(d_j\beta_\xi\big)^2\Big[1+d_j\beta_\xi\Big(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\Big)\Big]^2} \Bigg) +$$

$$\Bigg(\frac{\Big[1+d_j\beta_\xi\Big(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\Big)\Big]^{\frac{1}{d_j\beta_\xi}} \Big(d_j\beta_\xi\Big(\frac{d_j}{d_j\beta_\sigma}\Big)\Big)^2}{\big(d_j\beta_\xi\big)\Big[1+d_j\beta_\xi\Big(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\Big)\Big]^2} \Bigg)\Bigg) \Big[1+d_k\beta_\xi\Big(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\Big)\Big]^{\frac{2}{d_j\beta_\xi}} + 4\Bigg(a(h_{j,k})^2 \Bigg(\frac{\Big[1+d_j\beta_\xi\Big(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\Big)\Big]^{\frac{1}{d_j\beta_\xi}} \Big(d_j\beta_\xi\Big(\frac{d_j}{d_j\beta_\sigma}\Big)\Big)}{\big(d_j\beta_\xi\big)\Big[1+d_j\beta_\xi\Big(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\Big)\Big]} \Bigg) \times$$

$$\Bigg(\frac{\Big[1+d_k\beta_\xi\Big(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\Big)\Big]^{\frac{2}{d_j\beta_\xi}} d_k\beta_\xi \frac{d_k}{d_k\beta_\sigma}}{d_j\beta_\xi\Big[1+d_k\beta_\xi\Big(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\Big)\Big]} \Bigg)\Bigg) - 2\Bigg(a(h_{j,k})^2 \Big[1+d_j\beta_\xi\Big(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\Big)\Big]^{\frac{1}{d_j\beta_\xi}} \Bigg(-\frac{2\Big[1+d_k\beta_\xi\Big(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\Big)\Big]^{\frac{2}{d_j\beta_\xi}} \Big(d_k\beta_\xi\Big(\frac{d_k}{d_k\beta_\sigma}\Big)\Big)^2}{\big(d_j\beta_\xi\big)^2\Big[1+d_k\beta_\xi\Big(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\Big)\Big]^2} +$$

$$
\left.\left.\left.\left.\left.\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)^2}{\left(d_j\beta_\xi\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^2}\right)\right)\right)\right)\right) \Bigg/ \left(\left(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^2\right) \Bigg/
$$

$$
\left(\left(\left(\frac{\Phi(w)}{\left[1+d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}+\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(v)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}}\right)\times\right.\right.
$$

$$
\left(\frac{\Phi(v)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}}+\frac{\varphi(v)}{a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}\right)+
$$

$$
\frac{v\varphi(w)}{a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}-\frac{w\varphi(v)}{a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}\Bigg)-
$$

$$
\Bigg(\Bigg(\Bigg(\frac{2\Phi(w)\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}+\frac{2\varphi(w)\left(a(h_{j,k})\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\right)}{\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^2}+
$$

$$
\Bigg(\varphi(v)\Bigg(-a(h_{j,k})\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}-a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)}{d_k\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\Bigg)\Bigg)/
$$

$$\left.\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}\right)^2\right)\left(\frac{\Phi(v)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}}+\frac{\varphi(v)}{a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\right.$$

$$\left.\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}}\right)+\left(\frac{\Phi(w)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}+\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\right.$$

$$\left.\left.\frac{\varphi(v)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}}\right)\right)\left(\frac{2\Phi(v)\left(d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}d_k\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}+\right.$$

$$\frac{2\varphi(v)\left(a(h_{j,k})\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}} d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\right)}{\left(a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^2}+\left(\varphi(w)\left(-a(h_{j,k})\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\times\right.\right.$$

$$\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}-a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left.\left.\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}} d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\right)\right)\Big/\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\times\right.$$

$$\left.\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\right)^2\Bigg)-\left(v\varphi(w)\left(-2a(h_{j,k})^2\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}} d_j\beta_\xi\frac{d_j}{d_j\beta_\sigma}}{d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}-\right.\right.$$

$$a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}} d_k\beta_\xi\,\frac{d_k}{d_k\beta_\sigma}}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\Bigg)\Bigg)\Bigg) \Big/ \left(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\right)^2 +$$

$$\Bigg(v\varphi(w)\Bigg(-a(h_{j,k})^2\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}} - 2\Bigg(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\times$$

$$\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}} d_k\beta_\xi\,\frac{d_k}{d_k\beta_\sigma}}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\Bigg)\Bigg)\Bigg) \Big/ \left(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^2\Bigg) \Big/$$

$$
\left(\left(\left(\frac{\Phi(w)}{\left[1+d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}+\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(v)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}}\right)\times\right.\right.
$$

$$
\left(\frac{\Phi(v)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}}+\frac{\varphi(v)}{a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}\right)+
$$

$$
\left.\left.\frac{v\varphi(w)}{a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}-\frac{v\varphi(w)}{a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}\right)^2\right)
$$

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$$
\begin{aligned}
\frac{\partial^2 \ell(\boldsymbol{\beta})}{\partial\beta_\mu \partial\beta_\sigma} = \sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m} \Bigg(& -\frac{\Phi(w)\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\right)^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^2} + \frac{\Phi(w)\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)^2\right)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]} - \\
& \frac{\Phi(w)\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)\left[d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)}\right)\right]}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^2} - \frac{\Phi(v)\left(d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)\left[d_k\beta_\xi\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right]}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}\left(d_k\beta_\xi\right)^2\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^2} + \\
& \frac{\Phi(v)\left(d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)^2\right)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}\left(d_k\beta_\xi\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]} - \frac{\Phi(v)\left(d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)^2\right)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}\left(d_k\beta_\xi\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^2}\Bigg) \times
\end{aligned}
$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}\ln\left(\left(\left(\left(\frac{\Phi(w)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}+\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(v)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}}\right)\times\right.\right.\right.$$

$$\left.\left(\frac{\Phi(v)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}}+\frac{\varphi(v)}{a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}\right)+\right.$$

$$\left.\frac{v\varphi(w)}{a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}-\frac{w\varphi(v)}{a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}\right)+$$

$$\left(-\frac{\Phi(w)\left(d_j\beta_\xi \frac{x_{ji}}{d_j\beta_\sigma}\right)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}} d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]} - \frac{\Phi(v)\left(d_k\beta_\xi \frac{d_k}{d_k\beta_\sigma}\right)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}} d_k\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]} \right)\times$$

$$\left(\left(\left(\left(\frac{2\Phi(w)\left(d_j\beta_\xi \frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}} d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]} + \frac{2\varphi(w)\left[a(h_{j,k})\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}} d_j\beta_\xi \frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{d_j\beta_\sigma}}{d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\right]}{\left\{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right\}^2} + \right.\right.\right.\right.$$

$$\left[\varphi(v)\left(-a(h_{j,k})\left[\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}} d_j\beta_\xi \frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{d_j\beta_\sigma}}{d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right]\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}} - a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\times\right.\right.$$

$$\left[\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}} d_k\beta_\xi\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)}{d_k\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right]\Bigg)\Bigg] \Bigg/ \left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}\right)^2\Bigg)\times$$

$$\left(\frac{\Phi(v)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}}+\frac{\varphi(v)}{a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}\right)+$$

$$\left(\frac{\Phi(w)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}+\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(v)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}}\right)\times$$

$$\left(\frac{2\Phi(v)\left(d_k\beta_\xi\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}d_k\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}+\frac{2\varphi(v)a(h_{j,k})}{\left(a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^2}\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}+\right.$$

$$\left(\varphi(w)\left(-a(h_{j,k})\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}d_j\beta_\xi\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}}{d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}-a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}d_k\beta_\xi\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\right)\right)\right)/$$

$$\left.\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\right)^2\right)\right)-\left(v\varphi(w)\left(-2\left(a(h_{j,k})^2\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)}{d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\right)-\right.\right.\right.$$

$$a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}} d_k\beta_\xi\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{d_k\beta_\sigma}}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\Bigg)\Bigg)\Bigg/\left(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\right)^2+$$

$$\Bigg(w\varphi(v)\Bigg(-(a(h_{j,k})^2)\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}} d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)}{d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}-2\Bigg((a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\times$$

$$\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}} d_k\beta_\xi\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg/\left(\left(\left(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^2\right)\right)/$$

$$\left(\left(\frac{\Phi(w)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}+\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(v)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}}\right)\times\right.$$

$$\left(\frac{\Phi(v)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}}+\frac{\varphi(v)}{a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}\right)+$$

$$\left.\frac{v\varphi(w)}{a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}-\frac{w\varphi(v)}{a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}\right)\right)+$$

$$\left(-\frac{\Phi(w)\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}-\frac{\Phi(v)\left(d_k\beta_\xi\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}\left(d_k\beta_\xi\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\times$$

$$\left(\left(\left(\left(\frac{2\Phi(w)\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}+\frac{2\varphi(w)\left(a(h_{j,k})\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)}{\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^2}+\right.\right.\right.\right.$$

$$\left(\varphi(v)\left(-a(h_{j,k})\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}d_j\beta_\xi\frac{d_j}{\left(d_j\beta_\sigma\right)}}{d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}-a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}d_k\beta_\xi\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}}{d_k\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\right)\right)\Bigg)\Bigg)/$$

$$\left(a(h_{j,k}) \left[1 + d_j \beta_\xi \left(\frac{x_{ji} - d_j \beta_\mu}{d_j \beta_\sigma} \right) \right]^{\frac{1}{d_j \beta_\xi}} \left[1 + d_k \beta_\xi \left(\frac{x_{ki} - d_k \beta_\mu}{d_k \beta_\sigma} \right) \right]^{\frac{1}{d_k \beta_\xi}} \right)^2 \Bigg) \left(\frac{\Phi(v)}{\left[1 + d_k \beta_\xi \left(\frac{x_{ki} - d_k \beta_\mu}{d_k \beta_\sigma} \right) \right]^{\frac{2}{d_k \beta_\xi}}} + \frac{\varphi(v)}{a(h_{j,k}) \left[1 + d_k \beta_\xi \left(\frac{x_{ki} - d_k \beta_\mu}{d_k \beta_\sigma} \right) \right]^{\frac{2}{d_j \beta_\xi}}} - \right.$$

$$\left. \frac{\varphi(w)}{a(h_{j,k}) \left[1 + d_j \beta_\xi \left(\frac{x_{ji} - d_j \beta_\mu}{d_j \beta_\sigma} \right) \right]^{\frac{1}{d_j \beta_\xi}} \left[1 + d_k \beta_\xi \left(\frac{x_{ki} - d_k \beta_\mu}{d_k \beta_\sigma} \right) \right]^{\frac{1}{d_j \beta_\xi}}} \right) + \left(\frac{\Phi(w)}{\left[1 + d_j \left(\frac{x_{ji} - d_j \beta_\mu}{d_j \beta_\sigma} \right) \right]^{\frac{2}{d_j \beta_\xi}}} + \frac{\varphi(w)}{a(h_{j,k}) \left[1 + d_j \beta_\xi \left(\frac{x_{ji} - d_j \beta_\mu}{d_j \beta_\sigma} \right) \right]^{\frac{2}{d_j \beta_\xi}}} - \right.$$

$$\left. \frac{\varphi(v)}{a(h_{j,k}) \left[1 + d_j \beta_\xi \left(\frac{x_{ji} - d_j \beta_\mu}{d_j \beta_\sigma} \right) \right]^{\frac{1}{d_j \beta_\xi}} \left[1 + d_k \beta_\xi \left(\frac{x_{ki} - d_k \beta_\mu}{d_k \beta_\sigma} \right) \right]^{\frac{1}{d_k \beta_\xi}}} \right) \Bigg) \left(\frac{2\Phi(v) \left(d_k \beta_\xi \frac{d_k}{(d_k \beta_\sigma)} \right)}{\left[1 + d_k \beta_\xi \left(\frac{x_{ki} - d_k \beta_\mu}{d_k \beta_\sigma} \right) \right]^{\frac{2}{d_k \beta_\xi}} (d_k \beta_\xi) \left[1 + d_k \beta_\xi \left(\frac{x_{ki} - d_k \beta_\mu}{d_k \beta_\sigma} \right) \right]} + \right.$$

$$
\frac{2\varphi(v)\left(a(h_{j,k})\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}} d_k\beta_\xi\frac{d_k}{\left(d_k\beta_\sigma\right)}}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\right)}{\left(a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^2}+\left(\varphi(w)\left(-a(h_{j,k})\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}-\right.\right.
$$

$$
\left.\left.a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}} d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\right)\right)\Bigg/\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\right)^2\Bigg)-
$$

$$
\left(v\varphi(w)\left(-2a(h_{j,k})^2\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}-a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\times\right.\right.
$$

$$\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\Bigg)\Bigg)\Bigg) \Bigg/ \left(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\right)^2 +$$

$$\Big(w\varphi(v)\Big(-a(h_{j,k})^2\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}} -2\Bigg(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\times$$

$$\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\Bigg)\Bigg)\Bigg)\Bigg) \Bigg/ \left(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^2\Bigg) \Bigg/$$

$$\left(\frac{\Phi(w)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}} + \frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}} - \frac{\varphi(v)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}} \right)\times$$

$$\left(\frac{\Phi(v)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}} + \frac{\varphi(v)}{a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}} - \frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}} \right)+$$

$$\left.\left. \frac{v\varphi(w)}{a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}} - \frac{w\varphi(v)}{a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}} \right)\right)+$$

$$\left(-\frac{\Phi(w)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}} - \frac{\Phi(v)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}} \right)\Bigg)\Bigg(\Bigg(\Bigg(\Bigg(\frac{4\Phi(w)\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\right)^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^2} -$$

$$\frac{2\Phi(w)\left(d_j\beta_\xi\left(\frac{d_j^{\,2}}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]} + \frac{2\Phi(w)\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^2} +$$

$$\frac{1}{\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^3}\Bigg(8\varphi(w)\Bigg(a(h_{j,k})\Bigg(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\Bigg)\Bigg)\Bigg(a(h_{j,k})\Bigg(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\Bigg)\Bigg)\Bigg)\Bigg) +$$

$$
\frac{1}{\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^2}\left(2\varphi(w)\left(a(h_{j,k})\left(\frac{2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{d_j\beta_\sigma}\right)\right)\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^2}-\right.\right.\right.
$$

$$
\left.\left.\left.\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j^{\,2}}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}+\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{d_j\beta_\sigma}\right)\right)\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^2}\right)\right)\right)-
$$

$$
\left(2\varphi(v)\left(-a(h_{j,k})\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}-a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\times\right.\right.
$$

$$\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}} d_k\beta_\xi\,\frac{d_k}{\left(d_k\beta_\sigma\right)}}{d_k\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\Bigg)\Bigg)\Bigg(-a(h_{j,k})\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}} d_j\beta_\xi\,\frac{\left(x_{ji}-d_j\right)d_j}{\left(d_j\beta_\sigma\right)^2}}{d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}-$$

$$a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}} d_k\beta_\xi\,\frac{\left(x_{ki}-d_k\right)d_k}{\left(d_k\beta_\sigma\right)^2}}{d_k\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\Bigg)\Bigg)\Bigg)\Bigg/\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\right)^3+$$

$$\Bigg(\varphi(v)\Bigg(-a(h_{j,k})\Bigg(-\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\,\frac{\left(x_{ji}-d_j\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^2}-\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j^{\,2}}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}+$$

$$\left.\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\frac{\left(x_{ji}-d_j\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^2}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}+a(h_{j,k})\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\frac{\left(x_{ji}-d_j\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\times$$

$$\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}\left(d_k\beta_\xi\frac{\left(x_{ki}-d_k\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\left(d_k\beta_\xi\frac{d_k}{\left(d_k\beta_\sigma\right)}\right)}{\left(d_k\beta_\xi\right)^2\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^2}+a(h_{j,k})\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\frac{\left(x_{ji}-d_j\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}d_k\beta_\xi\frac{d_k}{\left(d_k\beta_\sigma\right)}}{d_k\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)-$$

$$a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(-\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}\left(d_k\beta_\xi\frac{\left(x_{ki}-d_k\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\left(d_k\beta_\xi\frac{d_k}{\left(d_k\beta_\sigma\right)}\right)}{\left(d_k\beta_\xi\right)^2\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^2}-\right.$$

$$\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}} d_k\beta_\xi \frac{d_k^{\,2}}{\left(d_k\beta_\sigma\right)^2}}{d_k\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}+\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}\left(d_k\beta_\xi\frac{\left(x_{ki}-d_k\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\left(d_k\beta_\xi\frac{d_k}{\left(d_k\beta_\sigma\right)}\right)}{\left(d_k\beta_\xi\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^2}\Bigg)\Bigg)\Bigg)\Bigg/\Bigg(a\left(h_{j,k}\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\times$$

$$\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}\Bigg)^2\Bigg)\left(\frac{\Phi(v)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}}+\frac{\varphi(v)}{a\left(h_{j,k}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(w)}{a\left(h_{j,k}\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}\right)+$$

$$\Bigg(\frac{2\Phi(w)\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}+\frac{2\varphi(w)\left(a\left(h_{j,k}\right)\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\right)}{\left(a\left(h_{j,k}\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^2}+$$

$$\Bigg(\varphi(v)\Bigg(-a(h_{j,k})\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}-a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\times$$

$$\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)}{d_k\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\Bigg)\Bigg)\Bigg/\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}\right)^2\Bigg(\frac{2\Phi(v)\left(d_k\beta_\xi\left(\frac{\left(x_{ki}-d_k\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}d_k\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}+$$

$$\frac{2\varphi(v)\left(a(h_{j,k})\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}d_k\beta_\xi\left(\frac{\left(x_{ki}-d_k\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\right)}{\left(a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^2}+\left(\varphi(w)\left(-a(h_{j,k})\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\right.\right.\times$$

$$
\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}-a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}d_k\beta_\xi\left(\frac{\left(x_{ji}-d_j\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\Bigg)\Bigg)\Bigg/\Bigg(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\times
$$

$$
\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\Bigg)^2\Bigg)+\left(\frac{2\Phi(w)\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}+\frac{2\varphi(w)\left(a(h_{j,k})\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\right)}{\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^2}+
$$

$$
\Bigg(\varphi(v)\left(-a(h_{j,k})\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}-a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\times
$$

$$\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}} d_k\beta_\xi\left(\frac{\left(x_{ki}-d_k\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)}{d_k\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\Bigg)\Bigg)\Bigg)/\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}\right)^2\Bigg)\times$$

$$\left(\frac{2\Phi(v)\left(d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}} d_k\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}+\frac{2\varphi(v)\left(a(h_{j,k})\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}} d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\right)}{\left(a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^2}+\right.$$

$$\left(\varphi(w)\left(-a(h_{j,k})\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}-a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\times\right.$$

$$\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}} d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\Bigg)\Bigg)\Bigg) / \left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\right)^2\Bigg) + \Bigg(\Bigg(\Bigg(\frac{\Phi(w)}{\left[1+d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}} +$$

$$\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}} - \frac{\varphi(v)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}}\Bigg)\Bigg(\frac{4\Phi(v)\left(d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)d_k\beta_\xi\left(\frac{(x_{ki}-d_k)d_k}{(d_k\beta_\sigma)^2}\right)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}(d_k\beta_\xi)^2\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^2} -$$

$$\frac{2\Phi(v)\left(d_k\beta_\xi\left(\frac{d_k^{\,2}}{(d_k\beta_\sigma)^2}\right)\right)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}} d_k\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]} + \frac{2\Phi(v)\left(d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)d_k\beta_\xi\left(\frac{(x_{ki}-d_k)d_k}{(d_k\beta_\sigma)^2}\right)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}} d_k\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^2} + \frac{1}{\left(a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^3} \times$$

$$\left(8\varphi(v)\left(a(h_{j,k})\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\right)\left(a(h_{j,k})\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}d_k\beta_\xi\left(\frac{\left(x_{ki}-d_k\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\right)\right)\right)+\frac{1}{\left(a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^2}\times$$

$$\left(2\varphi(v)\left(a(h_{j,k})\left(-\frac{2\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_k\beta_\xi\left(\frac{\left(x_{ki}-d_k\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)}{\left(d_j\beta_\xi\right)^2\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^2}-\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}d_k\beta_\xi\left(\frac{{d_k}^2}{\left(d_k\beta_\sigma\right)^2}\right)}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}+\right.\right.\right.$$

$$\left.\left.\left.\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_k\beta_\xi\left(\frac{\left(x_{ki}-d_k\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)}{\left(d_j\beta_\xi\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^2}\right)\right)\right)-\left(2\varphi(w)\left(-a(h_{j,k})\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\times\right.\right.$$

$$\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}-a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\Bigg(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\Bigg)\Bigg)\Bigg(-a(h_{j,k})\Bigg(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{(x_{ji}-d_j)d_j}{(d_j\beta_\sigma)^2}\right)\right)}{(d_j\beta_\xi)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\Bigg)\times$$

$$\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}-a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\Bigg(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}d_k\beta_\xi\left(\frac{(x_{ki}-d_k)d_k}{(d_k\beta_\sigma)^2}\right)}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\Bigg)\Bigg)\Bigg)\Bigg)\Big/\Bigg(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\times$$

$$\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\Bigg)^3+\Bigg(\varphi(w)\Bigg(-a(h_{j,k})\Bigg(-\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{(x_{ji}-d_j)d_j}{(d_j\beta_\sigma)^2}\right)\right)\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{(d_j\beta_\xi)^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^2}-\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j^{\,2}}{(d_j\beta_\sigma)^2}\right)\right)}{(d_j\beta_\xi)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}+$$

$$
\left.\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^2}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}+a\left(h_{j,k}\right)\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{\left(d_j\beta_\sigma\right)}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\times
$$

$$
\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_k\beta_\xi\left(\frac{\left(x_{ki}-d_k\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)+a\left(h_{j,k}\right)\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_k\beta_\xi\left(\frac{\left(x_{ki}-d_k\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)+
$$

$$
a\left(h_{j,k}\right)\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}d_k\beta_\xi\left(\frac{d_k}{\left(d_k\beta_\sigma\right)}\right)}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)-a\left(h_{j,k}\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\times
$$

$$\left(-\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_k\beta_\xi\left(\frac{(x_{ki}-d_k)d_k}{(d_k\beta_\sigma)^2}\right)\right)d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)}{(d_j\beta_\xi)^2\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^2}-\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}d_k\beta_\xi\left(\frac{d_k{}^2}{\left((d_k\beta_\sigma)^2\right)}\right)}{(d_j\beta_\xi)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}+\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_k\beta_\xi\left(\frac{(x_{ki}-d_k)d_k}{(d_k\beta_\sigma)^2}\right)\right)d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)}{(d_j\beta_\xi)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^2}\right)\Bigg)\Bigg)\Bigg)/$$

$$\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\right)^2\Bigg)+\Big(2v\varphi(w)\Big(-2a(h_{j,k})^2\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{(d_j\beta_\xi)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\times$$

$$\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\Bigg)-a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{(d_j\beta_\xi)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\Bigg)\Big(-2a(h_{j,k})^2\times$$

$$\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\Bigg)-a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}d_k\beta_\xi\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}}{\left(d_j\beta_\xi\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\Bigg)\Bigg)/$$

$$\left(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\right)^3-\Bigg(v\varphi(w)\Bigg(-2\Big(a(h_{j,k})^2\left(-\frac{2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^2}-\right.$$

$$\left.-\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j^{\,2}}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}+\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^2}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\Bigg)+$$

$$\Bigg(2\Big(a(h_{j,k})^2\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_k\beta_\xi\left(\frac{\left(x_{ki}-d_k\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\Bigg)\Bigg)+2\Big(a(h_{j,k})^2\times$$

$$\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\Bigg)-a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\times$$

$$\Bigg(-\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_k\beta_\xi\left(\frac{\left(x_{ki}-d_k\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)}{\left(d_j\beta_\xi\right)^2\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^2}-\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}d_k\beta_\xi\left(\frac{{d_k}^2}{\left(d_k\beta_\sigma\right)^2}\right)}{\left(d_j\beta_\xi\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}+$$

$$\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_k\beta_\xi\left(\frac{\left(x_{ki}-d_k\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)}{\left(d_j\beta_\xi\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^2}\Bigg)\Bigg)\Bigg)\Bigg/\left(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\right)^2-$$

$$\Big(2v\varphi(w)\Big(-\Big(a(h_{j,k})^2\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}-2\Bigg(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\times$$

$$\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\Bigg)\Bigg)\Bigg)\Big(-a(h_{j,k})^2\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}-$$

$$2\Bigg(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\Bigg(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_k\beta_\xi\left(\frac{\left(x_{ki}-d_k\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg/\Bigg(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\times$$

$$\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\Bigg)^3+\Big(v\varphi(w)\Big(-2\Big(a(h_{j,k})^2\Bigg(-\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^2}-$$

$$\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j^{\,2}}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}+\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^2}\Bigg)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}+$$

$$\left(2\left(a(h_{j,k})^2\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_k\beta_\xi\left(\frac{\left(x_{ki}-d_k\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\right)+2\left(a(h_{j,k})^2\times\right.\right.$$

$$\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_k\beta_\xi\left(\frac{d_k}{\left(d_k\beta_\sigma\right)}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\Bigg)-2\left(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\times\right.$$

$$\left(-\frac{2\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_k\beta_\xi\left(\frac{\left(x_{ki}-d_k\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)\left(d_k\beta_\xi\left(\frac{d_k}{\left(d_k\beta_\sigma\right)}\right)\right)}{\left(d_j\beta_\xi\right)^2\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^2}-\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_k\beta_\xi\left(\frac{d_k^{\,2}}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}+\right.$$

$$\left.\left.\left.\left.\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_k\beta_\xi\left(\frac{\left(x_{ki}-d_k\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)d_k\beta_\xi\left(\frac{d_k}{d_k\beta_\sigma}\right)}{\left(d_j\beta_\xi\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^2}\right)\right)\right)\right) \Bigg/ \left(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^2\Bigg) \Bigg/$$

$$\left(\frac{\Phi(w)}{\left[1+d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}+\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(v)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}}\right)\times$$

$$\left(\frac{\Phi(v)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}}+\frac{\varphi(v)}{a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}\right)+$$

$$\frac{v\varphi(w)}{a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}-\frac{v\varphi(w)}{a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}\Bigg)-$$

$$\Bigg(\Bigg(\frac{2\Phi(w)\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}+\frac{2\varphi(w)\left(a(h_{j,k})\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)}{\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^2}+$$

$$\left(\varphi(v)\left(-a(h_{j,k})\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\right)\right.\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}-a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\times$$

$$\Bigg(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}} d_k\beta_\xi\left(d_k\beta_\xi\frac{\left(x_{ki}-d_k\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)}{d_k\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\Bigg)\Bigg)\Bigg)\Bigg) / \Bigg(a(h_{j,k})\times\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}\Bigg)^2\Bigg)\Bigg(\frac{\Phi(v)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}}+$$

$$\frac{\varphi(v)}{a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}\Bigg)+\Bigg(\frac{\Phi(w)}{\left[1+d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}+\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-$$

$$\frac{\varphi(v)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}}\Bigg)\Bigg(\frac{2\Phi(v)\left(d_k\beta_\xi\frac{\left(x_{ki}-d_k\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}\left(d_k\beta_\xi\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}+$$

$$\frac{2\varphi(v)\left(a(h_{j,k})\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_k\beta_\xi\frac{\left(x_{ki}-d_k\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\right)}{\left(a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^2}+\Bigg(\varphi(w)\Bigg(-a(h_{j,k})\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\times$$

$$\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}-a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_k\beta_\xi\frac{\left(x_{ki}-d_k\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\Bigg)\Bigg)\Bigg)\Bigg/\Bigg(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\times$$

$$\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\Bigg)^2\Bigg)-\Bigg(v\varphi(w)\Bigg(-2a(h_{j,k})^2\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}-a(h_{j,k})^2\times$$

$$\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_k\beta_\xi\frac{\left(x_{ki}-d_k\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)}{\left(d_j\beta_\xi\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\Bigg)\Bigg)\Bigg/\left(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\right)^2+$$

$$\Bigg(w\varphi(v)\Bigg(-a(h_{j,k})^2\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}-2\Bigg(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\times$$

$$\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_k\beta_\xi\frac{\left(x_{ki}-d_k\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)}{\left(d_j\beta_\xi\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\Bigg)\Bigg)\Bigg)\Bigg/\left(\left(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^2\right)\times$$

$$\Bigg(\Bigg(\Bigg(\Bigg(\frac{2\Phi(w)\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}+\frac{2\varphi(w)\left(a(h_{j,k})\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)}{\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^2}+$$

$$\Bigg(\varphi(v)\Bigg(-a(h_{j,k})\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}-a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\times$$

$$\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}\left(d_k\beta_\xi\frac{d_k}{(d_k\beta_\sigma)}\right)}{d_k\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\Bigg)\Bigg)\Bigg)\Bigg)/\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}\right)^2\Bigg)\times$$

$$\left(\frac{\Phi(v)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}}+\frac{\varphi(v)}{a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}\right)+$$

$$\left(\frac{\Phi(w)}{\left[1+d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}+\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(v)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}}\right)\times$$

$$\left(\frac{2\Phi(v)\left(d_k\beta_\xi\frac{d_k}{(d_k\beta_\sigma)}\right)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}\left(d_k\beta_\xi\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}+\frac{2\varphi(v)\left(a(h_{j,k})\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_k\beta_\xi\frac{d_k}{(d_k\beta_\sigma)}\right)}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\right)}{\left(a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^2}+\right.$$

$$\left(\varphi(w)\left(-a(h_{j,k})\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}-a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\times\right.\right.$$

$$\left.\left.\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_k\beta_\xi\frac{d_k}{\left(d_k\beta_\sigma\right)}\right)}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\right)\right)\Bigg/\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\right)^2\Bigg)-$$

$$\left(v\varphi(w)\left(-2a(h_{j,k})^2\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}-a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\times\right.\right.$$

$$\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_k\beta_\xi\frac{d_k}{\left(d_k\beta_\sigma\right)}\right)}{\left(d_j\beta_\xi\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\Bigg)\Bigg)\Bigg)/\left(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\right)^2+\Big(v\varphi(w)\Big(-a(h_{j,k})^2\times$$

$$\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}-2\left(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\times\right.$$

$$\left.\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_k\beta_\xi\frac{d_k}{\left(d_k\beta_\sigma\right)}\right)}{\left(d_j\beta_\xi\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\right)\Bigg)\Bigg)\Bigg)/\left(\left(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^2\right)/$$

$$\left(\left(\frac{\Phi(w)}{\left[1+d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}+\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(v)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}}\right)\times\right.$$

$$\left(\frac{\Phi(v)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}}+\frac{\varphi(v)}{a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}\right)+$$

$$\left.\frac{v\varphi(w)}{a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}-\frac{w\varphi(v)}{a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}\right)$$

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$$
\frac{\partial^2 \ell(\boldsymbol{\beta})}{\partial\beta_\mu \partial\beta_\zeta} = \sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}\left(
\frac{\Phi(w)\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)\left(-\frac{d_j \ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}
+\frac{\Phi(w)\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)d_j}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\right)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)} -
\right.
$$

$$
\frac{\Phi(w)\left(d_j\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}
+\frac{\Phi(w)\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^2}+
$$

$$
\frac{\Phi(v)\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)\left(-\frac{d_k \ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}
+\frac{\Phi(v)\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)d_k}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\right)^2\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)} -
$$

$$\left.\frac{\Phi(v)\left(d_k\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}+\frac{\Phi(v)\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}\right)\times$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}\ln\left(\left(\left(\left(\frac{\Phi(w)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}+\frac{\varphi(w)}{a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(v)}{a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}}\right)\times\right.\right.\right.$$

$$\left(\frac{\Phi(v)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}+\frac{\varphi(v)}{a(h_{j,k})\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(w)}{a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}\right)+$$

$$\left.\frac{v\varphi(w)}{\left(a(h_{j,k})^{2}\right)\left(1+d_{j}\beta_{\zeta}\left(\frac{x_{ji}-d_{j}\beta_{\mu}}{d_{j}\beta_{\sigma}}\right)\right)^{\frac{2}{d_{j}\beta_{\zeta}}}\left(1+d_{k}\beta_{\zeta}\left(\frac{x_{ki}-d_{k}\beta_{\mu}}{d_{k}\beta_{\sigma}}\right)\right)^{\frac{1}{d_{j}\beta_{\zeta}}}}-\frac{w\varphi(v)}{\left(a(h_{j,k})^{2}\right)\left(1+d_{j}\beta_{\zeta}\left(\frac{x_{ji}-d_{j}\beta_{\mu}}{d_{j}\beta_{\sigma}}\right)\right)^{\frac{1}{d_{j}\beta_{\zeta}}}\left(1+d_{k}\beta_{\zeta}\left(\frac{x_{ki}-d_{k}\beta_{\mu}}{d_{k}\beta_{\sigma}}\right)\right)^{\frac{2}{d_{j}\beta_{\zeta}}}}\right)+$$

$$\left(\frac{\Phi(w)\left(d_{j}\beta_{\zeta}\left(\frac{d_{j}}{d_{j}\beta_{\sigma}}\right)\right)}{\left(1+d_{j}\beta_{\zeta}\left(\frac{x_{ji}-d_{j}\beta_{\mu}}{d_{j}\beta_{\sigma}}\right)\right)^{\frac{1}{d_{j}\beta_{\zeta}}}\left(d_{j}\beta_{\zeta}\right)\left(1+d_{j}\beta_{\zeta}\left(\frac{x_{ji}-d_{j}\beta_{\mu}}{d_{j}\beta_{\sigma}}\right)\right)}-\frac{\Phi(v)\left(d_{k}\beta_{\zeta}\left(\frac{d_{k}}{d_{k}\beta_{\sigma}}\right)\right)}{\left(1+d_{k}\beta_{\zeta}\left(\frac{x_{ki}-d_{k}\beta_{\mu}}{d_{k}\beta_{\sigma}}\right)\right)^{\frac{1}{d_{k}\beta_{\zeta}}}\left(d_{k}\beta_{\zeta}\right)\left(1+d_{k}\beta_{\zeta}\left(\frac{x_{ki}-d_{k}\beta_{\mu}}{d_{k}\beta_{\sigma}}\right)\right)}\right)\times$$

$$\left(\left(\left(\left(-\frac{\Phi(w)\left(-\frac{2d_{j}\ln\left(1+d_{j}\beta_{\zeta}\left(\frac{x_{ji}-d_{j}\beta_{\mu}}{d_{j}\beta_{\sigma}}\right)\right)}{\left(d_{j}\beta_{\zeta}\right)^{2}}+\frac{2\left(d_{j}\left(\frac{x_{ji}-d_{j}\beta_{\mu}}{d_{j}\beta_{\sigma}}\right)\right)}{d_{j}\beta_{\zeta}\left(1+d_{j}\beta_{\zeta}\left(\frac{x_{ji}-d_{j}\beta_{\mu}}{d_{j}\beta_{\sigma}}\right)\right)}\right)}{\left(1+d_{j}\beta_{\zeta}\left(\frac{x_{ji}-d_{j}\beta_{\mu}}{d_{j}\beta_{\sigma}}\right)\right)^{\frac{2}{d_{j}\beta_{\zeta}}}}-\left(a(h_{j,k})\left(1+d_{j}\beta_{\zeta}\left(\frac{x_{ji}-d_{j}\beta_{\mu}}{d_{j}\beta_{\sigma}}\right)\right)^{\frac{2}{d_{j}\beta_{\zeta}}}\right)^{2}\times\right.\right.\right.\right.$$

$$
\varphi(w)\Bigg(a(h_{j,k})\Bigg(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\Bigg(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\Bigg)\Bigg)\Bigg)\times
$$

$$
\frac{1}{\left(a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\right)^2}+\Bigg(\varphi(v)\Bigg(a(h_{j,k})\Bigg(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+
$$

$$
\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\Bigg)\Bigg)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}+a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\times
$$

$$\left(-\frac{d_k \ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\Bigg)\Bigg)\Bigg)\Bigg) / a(h_{j,k})\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\right)^2\times$$

$$\left(\frac{\Phi(v)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}+\frac{\varphi(v)}{a(h_{j,k})\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(w)}{a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}\right)+$$

$$\left(\frac{\Phi(w)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}+\frac{\varphi(w)}{a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}-\frac{\varphi(v)}{a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}}\right)\times$$

$$
\left(-\frac{\Phi(v)\left(-\frac{2d_k \ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2} + \frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}} - \varphi(v)\left(a(h_{j,k})\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)\times\right.\right.
$$

$$
\left.\left.\left(\left(-\frac{2d_j \ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2} + \frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\right)\right) \Big/ \left(a(h_{j,k})\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2 +
$$

$$
\left(\varphi(w)\left(a(h_{j,k})\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j \ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2} + \frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}} +\right.\right.
$$

$$a(h_{j,k})\Bigg(1+d_j\beta_\zeta\bigg(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\bigg)\Bigg)^{\frac{1}{d_j\beta_\zeta}}\Bigg(\Bigg(1+d_k\beta_\zeta\bigg(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\bigg)\Bigg)^{\frac{1}{d_j\beta_\zeta}}-\frac{d_j\ln\Bigg(1+d_k\beta_\zeta\bigg(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\bigg)\Bigg)}{\big(d_k\beta_\zeta\big)^2}+\frac{d_k\bigg(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\bigg)}{d_k\beta_\zeta\Bigg(1+d_k\beta_\zeta\bigg(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\bigg)\Bigg)}\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)/$$

$$\Bigg(\big(a(h_{j,k})\big)\Bigg(1+d_j\beta_\zeta\bigg(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\bigg)\Bigg)^{\frac{1}{d_j\beta_\zeta}}\Bigg(1+d_k\beta_\zeta\bigg(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\bigg)\Bigg)^{\frac{1}{d_j\beta_\zeta}}\Bigg)^2\Bigg)-\Bigg(v\varphi(w)\Bigg(\big(a(h_{j,k})^2\big)\Bigg(\Bigg(1+d_j\beta_\zeta\bigg(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\bigg)\Bigg)^{\frac{2}{d_j\beta_\zeta}}\Bigg(-\frac{2d_j\ln\Bigg(1+d_j\beta_\zeta\bigg(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\bigg)\Bigg)}{\big(d_j\beta_\zeta\big)^2}+$$

$$\frac{2\Bigg(d_j\bigg(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\bigg)\Bigg)}{d_j\beta_\zeta\Bigg(1+d_j\beta_\zeta\bigg(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\bigg)\Bigg)}\Bigg)\Bigg)\Bigg(1+d_k\beta_\zeta\bigg(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\bigg)\Bigg)^{\frac{1}{d_j\beta_\zeta}}+\big(a(h_{j,k})^2\big)\Bigg(1+d_j\beta_\zeta\bigg(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\bigg)\Bigg)^{\frac{2}{d_j\beta_\zeta}}\Bigg(\Bigg(1+d_k\beta_\zeta\bigg(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\bigg)\Bigg)^{\frac{1}{d_j\beta_\zeta}}\times$$

$$\left(-\frac{d_j \ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\Bigg)\Bigg)\Bigg)\Bigg)+\left(\left(a(h_{j,k})^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)^2+$$

$$\Bigg(w\varphi(v)\Bigg(\left(a(h_{j,k})^2\right)\Bigg(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j \ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\Bigg)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}+$$

$$\left(a(h_{j,k})^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(-\frac{2d_j \ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_{ki}\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)/$$

$$\left(\left(a(h_{j,k})^{2}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)^{2} \Bigg/ \left(\left(\left(\frac{\Phi(w)}{\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)}+\frac{\varphi(w)}{a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\right.\right.\right.$$

$$\left.\left.\frac{\varphi(v)}{a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}}\right)\right)\left(\left(\frac{\Phi(v)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}+\frac{\varphi(v)}{a(h_{j,k})\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\right.\right.$$

$$\left.\frac{\varphi(w)}{a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}\right)+\frac{v\varphi(w)}{\left(a(h_{j,k})^{2}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}-$$

$$\frac{w\varphi(v)}{\left(a(h_{j,k})^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}\Bigg)\Bigg)+\Bigg(\frac{\Phi(w)\left(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}+$$

$$\frac{\Phi(v)\left(-\frac{d_k\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}\Bigg)\Bigg)\Bigg(\Bigg(\Bigg(\Bigg(\Bigg(\frac{2\left(\Phi(w)\right)\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}+$$

$$
\frac{2\left(\varphi(w)\right)\left(a(h_{j,k})\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)}{\left(a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}+\varphi(v)\left(-a(h_{j,k})\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\times
$$

$$
\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}-a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)))\Bigg/\left(a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\right)^2\times
$$

$$
\left(\frac{\Phi(v)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}+\frac{\varphi(v)}{a(h_{j,k})\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(w)}{a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}\right)+
$$

$$\left(\frac{\Phi(w)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}+\frac{\varphi(w)}{a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(v)}{a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}}\right)\times$$

$$\left(\frac{2\big(\Phi(v)\big)\left(d_k\beta_\zeta\left(\frac{d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\big(d_k\beta_\zeta\big)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}+\frac{2\big(\varphi(v)\big)\left(a(h_{j,k})\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)}{\left(a(h_{j,k})\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}+\right.$$

$$\varphi(w)\Bigg(-a(h_{j,k})\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\Bigg)\times$$

$$-a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg/\left(a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)^2\Bigg)-v\varphi(w)\Bigg(-2\Bigg(\left(a(h_{j,k})^2\right)\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\times$$

$$\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg)-\left(a(h_{j,k})^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\Bigg)\times$$

$$
\frac{1}{\left(\left(a(h_{j,k})^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)^2}+\frac{1}{\left(\left(a(h_{j,k})^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}\times
$$

$$
\left(w\varphi(v)\left(-\left(a(h_{j,k})^2\right)\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}-2\left(\left(a(h_{j,k})^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\right)\right)/
$$

$$\left(\left(a(h_{j,k})^{2}\right)\left(1+d_{j}\beta_{\zeta}\left(\frac{x_{ji}-d_{j}\beta_{\mu}}{d_{j}\beta_{\sigma}}\right)\right)^{\frac{1}{d_{j}\beta_{\zeta}}}\left(1+d_{k}\beta_{\zeta}\left(\frac{x_{ki}-d_{k}\beta_{\mu}}{d_{k}\beta_{\sigma}}\right)\right)^{\frac{2}{d_{j}\beta_{\zeta}}}\right)^{2} \Bigg/ \Bigg(\Bigg(\Bigg(\frac{\Phi(w)}{\left(1+d_{j}\beta_{\zeta}\left(\frac{x_{ji}-d_{j}\beta_{\mu}}{d_{j}\beta_{\sigma}}\right)\right)^{\frac{2}{d_{j}\beta_{\zeta}}}}+\frac{\varphi(w)}{a(h_{j,k})\left(1+d_{j}\beta_{\zeta}\left(\frac{x_{ji}-d_{j}\beta_{\mu}}{d_{j}\beta_{\sigma}}\right)\right)^{\frac{2}{d_{j}\beta_{\zeta}}}}-$$

$$\frac{\varphi(v)}{a(h_{j,k})\left(1+d_{j}\beta_{\zeta}\left(\frac{x_{ji}-d_{j}\beta_{\mu}}{d_{j}\beta_{\sigma}}\right)\right)^{\frac{1}{d_{j}\beta_{\zeta}}}\left(1+d_{k}\beta_{\zeta}\left(\frac{x_{ki}-d_{k}\beta_{\mu}}{d_{k}\beta_{\sigma}}\right)\right)^{\frac{1}{d_{k}\beta_{\zeta}}}}\Bigg)\Bigg(\Bigg(\frac{\Phi(v)}{\left(1+d_{k}\beta_{\zeta}\left(\frac{x_{ki}-d_{k}\beta_{\mu}}{d_{k}\beta_{\sigma}}\right)\right)^{\frac{2}{d_{k}\beta_{\zeta}}}}+\frac{\varphi(v)}{a(h_{j,k})\left(1+d_{k}\beta_{\zeta}\left(\frac{x_{ki}-d_{k}\beta_{\mu}}{d_{k}\beta_{\sigma}}\right)\right)^{\frac{2}{d_{j}\beta_{\zeta}}}}-$$

$$\frac{\varphi(w)}{a(h_{j,k})\left(1+d_{j}\beta_{\zeta}\left(\frac{x_{ji}-d_{j}\beta_{\mu}}{d_{j}\beta_{\sigma}}\right)\right)^{\frac{1}{d_{j}\beta_{\zeta}}}\left(1+d_{k}\beta_{\zeta}\left(\frac{x_{ki}-d_{k}\beta_{\mu}}{d_{k}\beta_{\sigma}}\right)\right)^{\frac{1}{d_{j}\beta_{\zeta}}}}\Bigg)+\frac{v\varphi(w)}{\left(a(h_{j,k})^{2}\right)\left(1+d_{j}\beta_{\zeta}\left(\frac{x_{ji}-d_{j}\beta_{\mu}}{d_{j}\beta_{\sigma}}\right)\right)^{\frac{2}{d_{j}\beta_{\zeta}}}\left(1+d_{k}\beta_{\zeta}\left(\frac{x_{ki}-d_{k}\beta_{\mu}}{d_{k}\beta_{\sigma}}\right)\right)^{\frac{1}{d_{j}\beta_{\zeta}}}}-$$

$$
\left.\left.\frac{w\varphi(v)}{\left(a(h_{j,k})^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}\right)\right)+\left(-\frac{\Phi(w)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}-\frac{\Phi(v)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}}\right)\times
$$

$$
\left(\left(\left(-\frac{2\left(\Phi(w)\right)\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)\left(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}-2\left(\Phi(w)\right)\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)\times\right.\right.\right.
$$

$$
\frac{\left(-\frac{2d_j \ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}-\frac{2\left(\Phi(w)\right)\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)d_j}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\right)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}+
$$

$$
\frac{2\left(\Phi(w)\right)\left(d_j\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}-\frac{2\left(\Phi(w)\right)\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^2}-
$$

$$
\frac{4\left(\varphi(w)\right)\left(a(h_{j,k})\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\left(a(h_{j,k})\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}-\frac{2\left(1+d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\right)}{\left(a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^3}+
$$

$$\frac{1}{\left(a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}\Bigg(2\big(\varphi(w)\big)\Bigg(a(h_{j,k})\Bigg(\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)\right)/\,d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)\times$$

$$\left(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)-\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)d_j}{\left(d_j\beta_\zeta\right)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}+\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}-$$

$$\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^2}\Bigg)\Bigg)\Bigg)-\Big(2\varphi(v)\Big(-a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\Big)-$$

$$\Bigg(a(h_{j,k}) \Bigg(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}} \left(-\frac{d_j\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2} + \frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)} \right)\Bigg) \left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}} +$$

$$\Bigg(-\frac{d_k \ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta} + \frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)} \Bigg)\Bigg)\Bigg)\Bigg) + \frac{1}{\left(a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\right)^2}\times$$

$$\Bigg(-\frac{d_k \ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta} + \Bigg(\varphi(v) \Bigg(-a(h_{j,k}) \Bigg(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}} \left(-\frac{d_j \ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2} + \frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)} \right) \left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)} -$$

$$
\left.\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)d_j}{\left(d_j\beta_\zeta\right)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}+\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}-\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)\left(d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^2}\right)\times
$$

$$
\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}-a(h_{j,k})\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(-\frac{d_k\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+\right.\right.
$$

$$
\left.\left.\left.\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\right)-\left(a(h_{j,k})\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\right)\times
$$

$$\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)-a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg(\frac{\left(-\frac{d_k\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\times$$

$$\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)-\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)d_k}{\left(d_k\beta_\zeta\right)^2\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}+\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}-$$

$$-\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\Bigg)\Bigg)\Bigg)\Bigg/\left(\left(a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\right)^2\right)\times$$

$$\left(\frac{\Phi(v)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}+\frac{\varphi(v)}{a(h_{j,k})\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(w)}{a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}\right)+$$

$$\left(\frac{2\left(\Phi(w)\right)\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}+\frac{2\left(\varphi(w)\right)\left(a(h_{j,k})\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)}{\left(a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}+$$

$$\varphi(v)\left(-a(h_{j,k})\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}-a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\times$$

$$\frac{1}{\left(a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\right)^2}\left.\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{jk}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ji}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\Bigg)\Bigg)\times$$

$$\left(-\frac{\Phi(v)\left(-\frac{2d_k\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+\frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}-\frac{\varphi(v)\left(a(h_{j,k})\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(-\frac{2d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\right)}{\left(a(h_{j,k})\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}+\right.$$

$$\frac{1}{\left(a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)^2}\left(\varphi(w)\left(a(h_{j,k})\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\right.\right.\right.\right.\right.$$

$$
\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\Bigg)\Bigg)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}+a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg(-\frac{d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+
$$

$$
\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)+\Bigg(-\frac{\Phi(w)\left(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-
$$

$$
-\frac{\varphi(w)\left(a(h_{j,k})\left(\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\right)\right)}{\left(a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}+
$$

$$\frac{1}{\left(a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\right)^2}\left(\varphi(v)\left(a(h_{j,k})\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}{\left(d_j\beta_\zeta\right)^2}+\right.\right.\right.\right.$$

$$\left.\left.\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}+a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\times\right.$$

$$\left(-\frac{d_k\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\right)\right)\right)\left(\frac{2\left(\Phi(v)\right)\left(d_k\beta_\zeta\left(\frac{d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}+\frac{2\left(\varphi(v)\right)\left(a(h_{j,k})\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)}{\left(a(h_{j,k})\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}+\right.$$

$$\frac{\varphi(w)\left(-a(h_{j,k})\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}-a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)}{\left(a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)^2}+$$

$$\left(\frac{\Phi(w)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}+\frac{\varphi(w)}{a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(v)}{a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}}\right)\times$$

$$\left(-\frac{2\left(\Phi(v)\right)\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)\left(-\frac{2d_k\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+\frac{2\left(d_k\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}-\frac{2\left(\Phi(v)\right)\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)d_k}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\right)^2\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}+\right.$$

$$\frac{2\left(\Phi(v)\right)\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}-\frac{2\left(\Phi(v)\right)\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}-$$

$$\frac{4\left(\Phi(v)\right)\left(a(h_{j,k})\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\left(a(h_{j,k})\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(-\frac{2d_k\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+\frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\right)}{\left(a(h_{j,k})\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^3}+$$

$$\frac{1}{\left(a(h_{j,k})\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}\left(2\left(\varphi(v)\right)\left(a(h_{j,k})\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(-\frac{2d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+\frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}-\right.\right.\right.$$

$$\left.\left.\left.\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)d_j}{\left(d_k\beta_\zeta\right)^2\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}+\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}-\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}\right)\right)\right)-$$

$$\frac{1}{\left(a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)^3}\left(2\left(\varphi(w)\right)\left(-a(h_{j,k})\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}-\right.\right.\right.$$

$$\left.\left.a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\right)\left(a(h_{j,k})\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\right.\right.\right.$$

$$\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\Bigg)\Bigg)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}+a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg(-\frac{d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+$$

$$\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\Bigg)\Bigg)\Bigg)\Bigg)+\frac{1}{\left(a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)^2}\times$$

$$\Bigg(\varphi(w)\Bigg(-a(h_{j,k})\Bigg(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}-$$

$$\left.\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)d_j}{\left(d_j\beta_\zeta\right)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}+\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}-\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^2}\right)\times$$

$$\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}-a(h_{j,k})\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)-$$

$$a(h_{j,k})\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)-a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\times$$

$$
\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta}+\frac{d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}-\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)d_j}{\left(d_j\beta_\zeta\right)^2\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}+\right.
$$

$$
\left.\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}-\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}\right)\Bigg)\Bigg)\Bigg)\Bigg)+
$$

$$
\frac{1}{\left(\left(a(h_{j,k})^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)^3}\left(2\left(v\varphi(w)\right)\left(-2\left(\left(a(h_{j,k})^2\right)\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}-\right.\right.\right.\right.
$$

$$\left(a(h_{j,k})^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\Bigg(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\Bigg)\Bigg)\Bigg)\Bigg(\left(a(h_{j,k})^2\right)\Bigg(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\Bigg(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+$$

$$\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\Bigg)\Bigg)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}+\left(a(h_{j,k})^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\Bigg(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg(-\frac{d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+$$

$$\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\Bigg)\Bigg)\Bigg)\Bigg)-\frac{1}{\left(\left(a(h_{j,k})^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)^2}\Big(v\varphi(w)\Big(-2\Big(\left(a(h_{j,k})^2\right)\times$$

$$
\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}}-\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)d_j}{\left(d_j\beta_\zeta\right)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}+\right.
$$

$$
\left.\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}-\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^2}\right)\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)-
$$

$$
2\left(\left(a(h_{j,k})^2\right)\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\right.\right.
$$

$$\left.\left.\left.\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\right)-\left(a(h_{j,k})^2\right)\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\right.\right.$$

$$\left.\left.\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)-\left(a(h_{j,k})^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\times$$

$$\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}-\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)d_j}{\left(d_j\beta_\zeta\right)^2\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}+\right.$$

$$\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}-\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}\Bigg)\Bigg)\Bigg)-$$

$$\frac{1}{\left(\left(a(h_{j,k})^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^3}\left(2\left(w\varphi(v)\right)\left(-\left(a(h_{j,k})^2\right)\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}-\right.\right.$$

$$\left.2\left(\left(a(h_{j,k})^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\right)\right)\left(\left(a(h_{j,k})^2\right)\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\right.\right.\right.$$

$$\left.\left.\frac{d_j\left(\frac{x_{ii}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ii}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}+\left(a(h_{j,k})^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(-\frac{2d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\right.\right.$$

$$\left.\left.\left.\left.\left.\frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\right)\right)\right)+\frac{1}{\left(\left(a(h_{j,k})^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}\left(w\varphi(v)\left(-\left(a(h_{j,k})^2\right)\times\right.\right.$$

$$\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_j\left(\frac{x_{ii}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ii}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ii}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}-\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)d_j}{\left(d_j\beta_\zeta\right)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ii}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}+\right.$$

$$\left.\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ii}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}-\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ii}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^2}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}-$$

$$\left(a(h_{j,k})^2\right)\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(-\frac{2d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)-$$

$$2\left(\left(a(h_{j,k})^2\right)\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_j\left(\frac{x_{ii}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ii}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\left(\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)-$$

$$
2\Bigg(\left(a(h_{j,k})^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(-\frac{2d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\frac{\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}-
$$

$$
\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)d_k}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}+\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}-\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)/
$$

$$
\left(\frac{\Phi(w)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}+\frac{\varphi(w)}{a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(v)}{a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}}\right)\times
$$

$$\left(\frac{\Phi(v)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}+\frac{\varphi(v)}{a(h_{j,k})\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(w)}{a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}\right)+$$

$$\left.\frac{v\varphi(w)}{\left(a(h_{j,k})^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}-\frac{w\varphi(v)}{\left(a(h_{j,k})^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}\right)-$$

$$\left(\left(-\frac{\Phi(w)\left(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{1}{\left(a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}\times\right.\right.$$

$$\varphi(w)\Bigg(a(h_{j,k})\Bigg(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\Bigg(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\Bigg)\Bigg)\Bigg)+\frac{1}{\left(a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\right)^2}\times$$

$$\Bigg(\varphi(v)\Bigg(a(h_{j,k})\Bigg(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\Bigg)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}+$$

$$a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\Bigg(-\frac{d_k\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)\times$$

$$\left(\frac{\Phi(v)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}+\frac{\varphi(v)}{a(h_{j,k})\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(w)}{a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}\right)+$$

$$\left(\frac{\Phi(w)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}+\frac{\varphi(w)}{a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(v)}{a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}}\right)\times$$

$$\left(-\frac{\Phi(v)\left(-\frac{2d_k\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+\frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(v)\left(a(h_{j,k})\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(-\frac{2d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+\frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\right)}{\left(a(h_{j,k})\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}+\right.$$

$$\frac{1}{\left(a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)^2}\Bigg(\varphi(w)\Bigg(a(h_{j,k})\Bigg(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+$$

$$\frac{d_j\left(\frac{x_{ii}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ii}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\Bigg)\Bigg)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}+a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg(-\frac{d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+$$

$$\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)-\frac{1}{\left(\left(a(h_{j,k})^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)^2}\Bigg(v\varphi(w)\Bigg(\left(a(h_{j,k})^2\right)\Bigg(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\times$$

$$\left(-\frac{2d_j \ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\Bigg)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}+\left(a(h_{j,k})^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\times$$

$$\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j \ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\Bigg)\Bigg)+\frac{1}{\left(\left(a(h_{j,k})^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}\times$$

$$\left(w\varphi(v)\left(\left(a(h_{j,k})^2\right)\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j \ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_j\left(\frac{x_{ii}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ii}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}+\right.\right.$$

$$\left(a(h_{j,k})^{2}\right)\left(1+d_{j}\beta_{\zeta}\left(\frac{x_{ji}-d_{j}\beta_{\mu}}{d_{j}\beta_{\sigma}}\right)\right)^{\frac{1}{d_{j}\beta_{\zeta}}}\left(\left(1+d_{k}\beta_{\zeta}\left(\frac{x_{ki}-d_{k}\beta_{\mu}}{d_{k}\beta_{\sigma}}\right)\right)^{\frac{2}{d_{j}\beta_{\zeta}}}\left(-\frac{2d_{j}\ln\left(1+d_{k}\beta_{\zeta}\left(\frac{x_{ki}-d_{k}\beta_{\mu}}{d_{k}\beta_{\sigma}}\right)\right)}{\left(d_{j}\beta_{\zeta}\right)^{2}}+\frac{2\left(d_{k}\left(\frac{x_{ki}-d_{k}\beta_{\mu}}{d_{k}\beta_{\sigma}}\right)\right)}{d_{j}\beta_{\zeta}\left(1+d_{k}\beta_{\zeta}\left(\frac{x_{ki}-d_{k}\beta_{\mu}}{d_{k}\beta_{\sigma}}\right)\right)}\right)\right)\Bigg)\Bigg)\Bigg)\Bigg)\times$$

$$\Bigg(\Bigg(\Bigg(\Bigg(\frac{2\left(\Phi(w)\right)\left(d_{j}\beta_{\zeta}\left(\frac{d_{j}}{d_{j}\beta_{\sigma}}\right)\right)}{\left(1+d_{j}\beta_{\zeta}\left(\frac{x_{ji}-d_{j}\beta_{\mu}}{d_{j}\beta_{\sigma}}\right)\right)^{\frac{2}{d_{j}\beta_{\zeta}}}\left(d_{j}\beta_{\zeta}\right)\left(1+d_{j}\beta_{\zeta}\left(\frac{x_{ji}-d_{j}\beta_{\mu}}{d_{j}\beta_{\sigma}}\right)\right)}+\frac{2\left(\varphi(w)\right)\left(a(h_{j,k})\left(\frac{\left(1+d_{j}\beta_{\zeta}\left(\frac{x_{ji}-d_{j}\beta_{\mu}}{d_{j}\beta_{\sigma}}\right)\right)^{\frac{2}{d_{j}\beta_{\zeta}}}\left(d_{j}\beta_{\zeta}\left(\frac{d_{j}}{d_{j}\beta_{\sigma}}\right)\right)}{\left(1+d_{j}\beta_{\zeta}\left(\frac{x_{ji}-d_{j}\beta_{\mu}}{d_{j}\beta_{\sigma}}\right)\right)}\right)\right)}{\left(a(h_{j,k})\left(1+d_{j}\beta_{\zeta}\left(\frac{x_{ji}-d_{j}\beta_{\mu}}{d_{j}\beta_{\sigma}}\right)\right)^{\frac{2}{d_{j}\beta_{\zeta}}}\right)}+$$

$$\varphi(v)\left(-a(h_{j,k})\left(\frac{\left(1+d_{j}\beta_{\zeta}\left(\frac{x_{ji}-d_{j}\beta_{\mu}}{d_{j}\beta_{\sigma}}\right)\right)^{\frac{1}{d_{j}\beta_{\zeta}}}\left(d_{j}\beta_{\zeta}\left(\frac{d_{j}}{d_{j}\beta_{\sigma}}\right)\right)}{d_{j}\beta_{\zeta}\left(1+d_{j}\beta_{\zeta}\left(\frac{x_{ji}-d_{j}\beta_{\mu}}{d_{j}\beta_{\sigma}}\right)\right)}\right)\left(1+d_{k}\beta_{\zeta}\left(\frac{x_{ki}-d_{k}\beta_{\mu}}{d_{k}\beta_{\sigma}}\right)\right)^{\frac{1}{d_{k}\beta_{\zeta}}}-a(h_{j,k})\left(1+d_{j}\beta_{\zeta}\left(\frac{x_{ji}-d_{j}\beta_{\mu}}{d_{j}\beta_{\sigma}}\right)\right)^{\frac{1}{d_{j}\beta_{\zeta}}}\times\right.$$

$$\left(\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\Bigg/\left(a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)^2\right)\times$$

$$\left(\frac{\Phi(v)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}+\frac{\varphi(v)}{a(h_{j,k})\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(w)}{a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}\right)+$$

$$\left(\frac{\Phi(w)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}+\frac{\varphi(w)}{a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(v)}{a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}}\right)\times$$

$$\left(\frac{2\left(\varphi(v)\right)\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}+\frac{2\left(\varphi(v)\right)\left(a(h_{j,k})\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)}{\left(a(h_{j,k})\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}+\right.$$

$$\left.\frac{\varphi(w)\left(-a(h_{j,k})\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}-a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)}{\left(a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)^2}\right)-$$

$$\frac{v\varphi(w)\Bigg(-2\Bigg(\left(a(h_{j,k})^{2}\right)\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{d_j}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}-\left(\left(a(h_{j,k})^{2}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\Bigg)\Bigg)}{\left(\left(a(h_{j,k})^{2}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)^{2}}\Bigg) /$$

$$\Bigg(\Bigg(\Bigg(\frac{\Phi(w)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}+\frac{\varphi(w)}{a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(v)}{a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}}\Bigg)\times$$

$$\left(\frac{\Phi(v)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}+\frac{\varphi(v)}{a(h_{j,k})\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(w)}{a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}\right)+$$

$$\left(\frac{v\varphi(w)}{\left(a(h_{j,k})^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}-\frac{w\varphi(v)}{\left(a(h_{j,k})^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}\right)^2\Bigg)$$

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$$\frac{\partial^2 \ell(\boldsymbol{\beta})}{\partial\beta_\sigma \partial\beta_\sigma} = \sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}\left(-\frac{\Phi(w)\left(d_j\beta_\zeta\left(\frac{\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)^2}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\right)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^2} + \frac{2\Phi(w)\left(d_j\beta_\zeta\left(\frac{\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)d_j^{\,2}}{\left(d_j\beta_\sigma\right)^3}\right)\right)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)} - \right.$$

$$\frac{\Phi(w)\left(d_j\beta_\zeta\left(\frac{\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)^2}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^2} - \frac{\Phi(v)\left(d_k\beta_\zeta\left(\frac{\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)d_k}{\left(d_k\beta_\zeta\right)^2}\right)\right)^2}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\right)^2\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2} +$$

$$\frac{2\Phi(v)\left(d_k\beta_\zeta\left(\frac{\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)d_k{}^2}{\left(d_k\beta_\zeta\right)^3}\right)\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}-\frac{\Phi(v)\left(d_k\beta_\zeta\left(\frac{\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)d_k}{\left(d_k\beta_\zeta\right)^2}\right)\right)^2}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}\times$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}\ln\left(\left(\left(\left(\frac{\Phi(w)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}+\frac{\varphi(w)}{a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(v)}{a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}}\right)\times\right.\right.\right.$$

$$\left(\frac{\Phi(v)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}+\frac{\varphi(v)}{a(h_{j,k})\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(w)}{a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}\right)+$$

$$\frac{v\varphi(w)}{\left(a(h_{j,k})^{2}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}-\frac{w\varphi(v)}{\left(a(h_{j,k})^{2}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}\Bigg)+$$

$$2\Bigg(\Bigg(-\frac{\Phi(w)\left(d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{\left(d_j\beta_\sigma\right)^{2}}d_j\right)\right)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}-\frac{\Phi(v)\left(d_k\beta_\zeta\left(\frac{d_k}{d_k\beta_\sigma}\right)\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\Bigg)\times$$

$$\Bigg(\Bigg(\Bigg(\Bigg(\frac{2\Phi(w)\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)}{\left(d_j\beta_\sigma\right)^{2}}d_j\right)\right)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}+\frac{2\varphi(w)\left(a(h_{j,k})\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{\left(d_j\beta_\sigma\right)^{2}}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)}{\left(a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^{2}}+$$

$$\left(\varphi(v)\left(-a(h_{j,k})\frac{\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)\right)}{\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}-a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\times\right.\right.$$

$$\left.\left.\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)d_k}{\left(d_k\beta_\zeta\right)^2}\right)\right)}{\left(d_k\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\right)\Bigg/\left(a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\right)^2\right)\times$$

$$\left(\frac{\Phi(v)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}+\frac{\varphi(v)}{a(h_{j,k})\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(w)}{a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}\right)+$$

$$\left(\frac{\Phi(w)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}+\frac{\varphi(w)}{a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(v)}{a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}}\right)\times$$

$$\left(\frac{2\Phi(v)\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}+2\varphi(v)\left(a(h_{j,k})\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\right)\times$$

$$\frac{1}{\left(a(h_{j,k})\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}+\left(\varphi(w)\left(-a(h_{j,k})\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)}{\left(d_j\beta_\sigma\right)^2}d_j\right)\right)}{\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}-\right.$$

$$a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\Bigg)\Bigg)\Bigg) \Big/ a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\times$$

$$\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg)^2\Bigg)\Bigg)-\Bigg(v\varphi(w)\Bigg(-2\Bigg(a(h_{j,k})^2\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)}{d_j\beta_\xi\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\Bigg)-$$

$$a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(\frac{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}d_k\beta_\xi\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}}{d_j\beta_\xi\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]}\right)\Bigg)\Bigg)\Bigg/\Bigg(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\times$$

$$\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\Bigg)^2\Bigg)\Bigg)\Bigg/\Bigg(\Bigg(\Bigg(\frac{\Phi(w)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}+\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(v)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}}\Bigg)\times$$

$$\left(\frac{\Phi(v)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}}+\frac{\varphi(v)}{a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}\right)+$$

$$\Bigg(\Bigg(\Bigg(\Bigg(\frac{4\Phi(w)\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)^2}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\right)^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^2}-\frac{4\Phi(w)\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j^{\,2}}{\left(d_j\beta_\sigma\right)^3}\right)\right)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}+$$

$$\frac{2\Phi(w)\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)^2}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^2}+\frac{8\varphi(w)\left(a(h_{j,k})\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\right)^2}{\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^3}+$$

$$\frac{1}{\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^2}\left(2\varphi(w)\left(a(h_{j,k})\left(-\frac{2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)^2}{\left(d_j\beta_\xi\right)^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^2}-\frac{2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\times\right.\right.\right.$$

$$\left.\left.\left.\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j^{\,2}}{\left(d_j\beta_\sigma\right)^3}\right)\right)+\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)^2}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^2}\right)\right)\right)-\left(2\varphi(v)\left(-a(h_{j,k})\left(\frac{\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)\right)^{\frac{1}{d_j\beta_\zeta}}}{\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\times\right.\right.\right.$$

$$
\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}} a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\Bigg(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)}{\left(d_k\beta_\sigma\right)^2}d_k\right)\right)}{\left(d_k\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\Bigg)\Bigg)\Bigg)^2\Bigg)/
$$

$$
\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\right)^3+\Bigg(\varphi(v)\Bigg(-a(h_{j,k})\Bigg(-\frac{\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)\right)^2}{\left(d_j\beta_\zeta\right)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^2}-
$$

$$
\frac{2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j^{\,2}}{\left(d_j\beta_\sigma\right)^3}\right)\right)}{\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}+\frac{\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)\right)^2}{\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^2}\Bigg)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}+
$$

$$
2\left(a(h_{j,k})\left(\left(\frac{\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)\right)}{\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)}{\left(d_k\beta_\sigma\right)^2}d_k\right)\right)}{\left(d_k\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right.\right. -
$$

$$
a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)}{\left(d_k\beta_\sigma\right)^2}d_k\right)\right)^2}{\left(d_k\beta_\zeta\right)^2\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}-\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)}{\left(d_k\beta_\sigma\right)^3}d_k^{\,2}\right)\right)}{\left(d_k\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}+\right.
$$

$$
\left.\left.\left.\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)}{\left(d_k\beta_\sigma\right)^2}d_k\right)\right)^2}{\left(d_k\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}\right)\right)\right) \Big/ \left(a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\right)^2\right)\times
$$

$$\left(\frac{\Phi(v)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}}+\frac{\varphi(v)}{a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}\right)+$$

$$2\left(\left(\frac{2\Phi(w)\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}+\frac{2\varphi(w)\left(a(h_{j,k})\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\right)}{\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^2}+\right.\right.$$

$$\left(\varphi(v)\left(-a(h_{j,k})\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right.\right.\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}-a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\times$$

$$\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)}{\left(d_k\beta_\sigma\right)^2}d_k\right)\right)}{\left(d_k\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\Bigg)\Bigg)\Bigg)\Bigg/\Bigg(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}\Bigg)^2\Bigg)\times$$

$$\Bigg(\frac{2\Phi(v)\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}+\frac{2\varphi(v)\left(a(h_{j,k})\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)}{\left(a(h_{j,k})\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}+$$

$$\Bigg(\varphi(w)\Bigg(-a(h_{j,k})\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)}{\left(d_j\beta_\sigma\right)^2}d_j\right)\right)}{\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\Bigg)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}-a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\times$$

$$\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\Bigg)\Bigg)\Bigg) / a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg)^2\Bigg)+$$

$$\left(\frac{\Phi(w)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}+\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(v)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}}\right)\times$$

$$\left(\frac{4\Phi(v)\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)^2}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\right)^2\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}-\frac{4\Phi(v)\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k^{\,2}}{\left(d_k\beta_\sigma\right)^3}\right)\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}+\right.$$

$$\frac{2\Phi(v)\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)^2}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}+\frac{8\varphi(v)\left(a(h_{j,k})\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)^2}{\left(a(h_{j,k})\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^3}+$$

$$\frac{1}{\left(a(h_{j,k})\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}\Bigg(2\varphi(v)\Bigg(a(h_{j,k})\Bigg(-\frac{2\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)^2}{\left(d_j\beta_\zeta\right)^2\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}-\frac{2\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k^{\,2}}{\left(d_k\beta_\sigma\right)^3}\right)\right)}{\left(d_j\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}+$$

$$\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)^2}{\left(d_j\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}\Bigg)\Bigg)\Bigg)\Bigg)-\Bigg(2\varphi(w)\Bigg(-a(h_{j,k})\Bigg(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)}{\left(d_j\beta_\sigma\right)^2}d_j\right)\right)}{\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\Bigg)\times$$

$$\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}-a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)^2\Bigg)/$$

$$\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}\right)^3+\left(\varphi(w)\left(-a(h_{j,k})\left(-\frac{\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)}{\left(d_j\beta_\sigma\right)^2}d_j\right)\right)\right)^2}{\left(d_j\beta_\zeta\right)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^2}-\right.\right.\right.$$

$$\left.\frac{2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)}{\left(d_j\beta_\sigma\right)^3}d_j^{\,2}\right)\right)}{\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}+\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)}{\left(d_j\beta_\sigma\right)^2}d_j\right)\right)^2}{\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^2}\right)\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}+$$

$$2\Bigg(a(h_{j,k})\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)}{\left(d_j\beta_\sigma\right)^2}d_j\right)\right)}{\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\Bigg)-$$

$$a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\Bigg(-\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)^2}{\left(d_j\beta_\zeta\right)^2\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}-\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k^{\,2}}{\left(d_k\beta_\sigma\right)^3}\right)\right)}{\left(d_j\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}+$$

$$\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)^2}{\left(d_j\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}\Bigg)\Bigg)\Bigg)\Bigg)\Bigg/\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)^2\Bigg)+$$

$$
\Bigg(2v\varphi(w)\Bigg(-2\left(a(h_{j,k})\right)^{2}\Bigg(\frac{\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^{2}}\right)\right)\right)}{\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\Bigg)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg)-\Bigg(a(h_{j,k})^{2}\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\times
$$

$$
\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^{2}}\right)\right)}{\left(d_j\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\Bigg)\Bigg)^{2}\Bigg)\Bigg/\left(a(h_{j,k})^{2}\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)^{3}-
$$

$$
\Bigg(v\varphi(w)\Bigg(-2\left(a(h_{j,k})\right)^{2}\Bigg(-\frac{2\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^{2}}\right)\right)\right)^{2}}{\left(d_j\beta_\zeta\right)^{2}\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{2}}-\frac{2\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right.}{\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)}{\left(d_j\beta_\sigma\right)^{3}}d_j^{\,2}\right)\right)+
$$

$$\left(\frac{\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)\right)^2}{\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^2}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg)+4\Big(a(h_{j,k})^2\left(-\frac{2\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)\right)}{\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\times$$

$$\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{\left(d_j\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\Bigg)\Bigg)-a(h_{j,k})^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\Bigg(-\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)^2}{\left(d_j\beta_\zeta\right)^2\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}-$$

$$\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k^{\,2}}{\left(d_k\beta_\sigma\right)^3}\right)\right)}{\left(d_j\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}+\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)^2}{\left(d_j\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}\Bigg)\Bigg)\Bigg)\Bigg)\Bigg(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\times$$

$$\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg)^2 - \Bigg(2v\varphi(w)\Bigg(-a(h_{j,k})^2\Bigg(-\frac{\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)\right)}{\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\Bigg)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}} -$$

$$2\Bigg(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{\left(d_j\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\Bigg)\Bigg)\Bigg)^2\Bigg) / \Bigg(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\times$$

$$\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\Bigg)^3 + \Bigg(v\varphi(w)\Bigg(-a(h_{j,k})^2\Bigg(-\frac{\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)\right)^2}{\left(d_j\beta_\zeta\right)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^2} - \frac{2\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right.}{\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\times$$

$$\left(d_j\beta_\zeta \left(\frac{\left(x_{ji}-d_j\beta_\mu\right)}{\left(d_j\beta_\sigma\right)^3} d_j^{\,2} \right)\right) + \left.\frac{\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)\right)^2}{\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^2} \right) \left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}} + 4\left(a(h_{j,k})^2 \times\right.$$

$$\left(-\frac{\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)\right)}{\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\left.\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{\left(d_j\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right) - 2\left(a(h_{j,k})^2 \times\right.$$

$$\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}} \left(-\frac{2\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)^2}{\left(d_j\beta_\zeta\right)^2\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2} - \frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k^{\,2}}{\left(d_k\beta_\sigma\right)^3}\right)\right)}{\left(d_j\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)} + \right.$$

$$\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)^2}{\left(d_j\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}\Bigg)\Bigg)\Bigg)\Bigg) \Bigg/ \left(a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2\Bigg) \Bigg/$$

$$\left(\frac{\Phi(w)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}+\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(v)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}}\right)\times$$

$$\left(\frac{\Phi(v)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}}+\frac{\varphi(v)}{a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}\right)+$$

$$\frac{v\varphi(w)}{a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}-\frac{v\varphi(w)}{a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}\Bigg)-$$

$$\Bigg(\Bigg(\Bigg(\Bigg(\frac{2\Phi(w)\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}+\frac{2\varphi(w)\left(a(h_{j,k})\left(\frac{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{\left(d_j\beta_\xi\right)\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]}\right)\right)}{\left(a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^2}+$$

$$\Bigg(\varphi(v)\Bigg(-a(h_{j,k})\left(\frac{\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\xi\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)\right)}{\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}-a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\times$$

$$\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{\left(d_k\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\Bigg)\Bigg)\Bigg) \Bigg/ \left(a(h_{j,k})\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\right)^2\Bigg)\times$$

$$\left(\frac{\Phi(v)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}}+\frac{\varphi(v)}{a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}\right)+$$

$$\left(\frac{\Phi(w)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}+\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(v)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}}\right)\times$$

$$
\Bigg(\frac{2\Phi(v)\left(d_k \beta_\zeta \left(\frac{\left(x_{ki} - d_k \beta_\mu\right) d_k}{\left(d_k \beta_\sigma\right)^2} \right) \right)}{\left(1 + d_k \beta_\zeta \left(\frac{x_{ki} - d_k \beta_\mu}{d_k \beta_\sigma} \right) \right)^{\frac{2}{d_k \beta_\zeta}} \left(d_k \beta_\zeta \right) \left(1 + d_k \beta_\zeta \left(\frac{x_{ki} - d_k \beta_\mu}{d_k \beta_\sigma} \right) \right)} + \frac{2\varphi(v)\left(a(h_{j,k}) \left(\frac{\left(1 + d_k \beta_\zeta \left(\frac{x_{ki} - d_k \beta_\mu}{d_k \beta_\sigma} \right) \right)^{\frac{2}{d_j \beta_\zeta}} \left(d_k \beta_\zeta \left(\frac{\left(x_{ki} - d_k \beta_\mu\right) d_k}{\left(d_k \beta_\sigma\right)^2} \right) \right)}{d_j \beta_\zeta \left(1 + d_k \beta_\zeta \left(\frac{x_{ki} - d_k \beta_\mu}{d_k \beta_\sigma} \right) \right)} \right) \right)}{\left(a(h_{j,k}) \left(1 + d_k \beta_\zeta \left(\frac{x_{ki} - d_k \beta_\mu}{d_k \beta_\sigma} \right) \right)^{\frac{2}{d_j \beta_\zeta}} \right)^2} +
$$

$$
\Bigg(\varphi(w) \Bigg(-a(h_{j,k}) \left(\frac{\left(1 + d_j \beta_\zeta \left(\frac{x_{ji} - d_j \beta_\mu}{d_j \beta_\sigma} \right) \right)^{\frac{1}{d_j \beta_\zeta}} \left(d_j \beta_\zeta \left(\frac{\left(x_{ji} - d_j \beta_\mu\right)}{\left(d_j \beta_\sigma\right)^2} d_j \right) \right)}{\left(d_j \beta_\zeta \right) \left(1 + d_j \beta_\zeta \left(\frac{x_{ji} - d_j \beta_\mu}{d_j \beta_\sigma} \right) \right)} \right) \left[1 + d_k \beta_\xi \left(\frac{x_{ki} - d_k \beta_\mu}{d_k \beta_\sigma} \right) \right]^{\frac{1}{d_j \beta_\xi}} - a(h_{j,k}) \left[1 + d_j \beta_\xi \left(\frac{x_{ji} - d_j \beta_\mu}{d_j \beta_\sigma} \right) \right]^{\frac{1}{d_j \beta_\xi}} \times
$$

$$
\left(\frac{\left(1 + d_k \beta_\zeta \left(\frac{x_{ki} - d_k \beta_\mu}{d_k \beta_\sigma} \right) \right)^{\frac{1}{d_j \beta_\zeta}} \left(d_k \beta_\zeta \left(\frac{\left(x_{ki} - d_k \beta_\mu\right) d_k}{\left(d_k \beta_\sigma\right)^2} \right) \right)}{\left(d_j \beta_\zeta \right) \left(1 + d_k \beta_\zeta \left(\frac{x_{ki} - d_k \beta_\mu}{d_k \beta_\sigma} \right) \right)} \right) \Bigg) \Bigg) \Bigg) / \left(a(h_{j,k}) \left(1 + d_j \beta_\zeta \left(\frac{x_{ji} - d_j \beta_\mu}{d_j \beta_\sigma} \right) \right)^{\frac{1}{d_j \beta_\zeta}} \left[1 + d_k \beta_\xi \left(\frac{x_{ki} - d_k \beta_\mu}{d_k \beta_\sigma} \right) \right]^{\frac{1}{d_j \beta_\xi}} \right)^2 \Bigg) -
$$

$$\left(v\varphi(w) \left(-2a(h_{j,k})^2 \left(\frac{ \left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}} \left(d_j\beta_\xi \left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2} \right)\right)\right) }{ \left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)} \right) \left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}} - a(h_{j,k})^2 \left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}} \times \right.\right.$$

$$\left.\left.\left(\frac{ \left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}} \left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{\left(d_j\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)} \right)\right)\right) \Bigg/ \left(a(h_{j,k})^2 \left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}} \left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}} \right)^2 +$$

$$\left(w\varphi(v) \left(-a(h_{j,k})^2 \left(\frac{ \left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}} \left(d_j\beta_\xi \left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2} \right)\right)\right) }{ \left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)} \right) \left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}} - 2\left(a(h_{j,k})^2 \left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}} \times \right.\right.\right.$$

$$\Bigg(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg/\left(a(h_{j,k})^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\right)^2\Bigg)^2\Bigg/$$

$$\Bigg(\Bigg(\Bigg(\frac{\Phi(w)}{\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}+\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(v)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_k\beta_\xi}}}\Bigg)\times$$

$$\Bigg(\frac{\Phi(v)}{\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_k\beta_\xi}}}+\frac{\varphi(v)}{a(h_{j,k})\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}-\frac{\varphi(w)}{a(h_{j,k})\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}\Bigg)+$$

$$
\left.\left.\frac{v\varphi(w)}{a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}}-\frac{v\varphi(w)}{a(h_{j,k})^2\left[1+d_j\beta_\xi\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right]^{\frac{1}{d_j\beta_\xi}}\left[1+d_k\beta_\xi\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right]^{\frac{2}{d_j\beta_\xi}}}\right)^2\right)
$$

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$$
\frac{\partial^2}{\partial\beta_\sigma\partial\beta_\zeta}A=\left(\frac{\Phi(w)\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)\left(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}+\frac{\Phi(w)\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)d_j}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\right)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}-\right.
$$

$$
\frac{\Phi(w)\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)d_j}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}+\frac{\Phi(w)\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)\left(d_j\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)}{d_j\beta_\zeta}\right)\right)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^2}+
$$

$$\frac{\Phi(v)\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)\left(-\frac{d_k\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}+\frac{\Phi(v)\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)d_k}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\right)^2\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}-$$

$$\left.\frac{\Phi(v)\left(d_k\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}+\frac{\Phi(v)\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)\left(d_k\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)}{d_k\beta_\sigma}\right)\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}\right)\times$$

$$\ln\left(\left(\left(\left(\frac{\Phi(w)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}+\frac{\varphi(w)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(v)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}}\right)\right.\right.\right.\times$$

$$\left(\frac{\Phi(v)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}+\frac{\varphi(v)}{a\left(h_{j,k}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(w)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}\right)+$$

$$\frac{v\varphi(w)}{\left(a\left(h_{j,k}\right)^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}-\frac{w\varphi(v)}{\left(a\left(h_{j,k}\right)^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}\Bigg)+$$

$$\frac{\Phi(w)\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}-\frac{\Phi(v)\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\Bigg)\times$$

$$\left(\left(\left(\left(-\frac{\Phi(w)\left(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{1}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}\times\right.\right.\right.\right.$$

$$\left.\left.\left.\varphi(w)\left(a\left(h_{j,k}\right)\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\right)\right)\right)+$$

$$\frac{1}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\right)^2}\left(\varphi(v)\left(a\left(h_{j,k}\right)\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\right.\right.\right.\right.$$

$$\left.\left.\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}+a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\times\right.$$

$$\left.\left.\left.\left.\left.\left(-\frac{d_k\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\right)\right)\right)\Bigg/\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\right)^2\right)\times$$

$$\left(\frac{\Phi(v)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}+\frac{\varphi(v)}{a\left(h_{j,k}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(w)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}\right)+$$

$$
\left(\frac{\Phi(w)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}+\frac{\varphi(w)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(v)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}}\right)\times
$$

$$
\left(-\frac{\Phi(v)\left(-\frac{2d_k\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+\frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}-\frac{1}{\left(a\left(h_{j,k}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}\times\right.
$$

$$
\varphi(v)\Bigg(a\left(h_{j,k}\right)\Bigg(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\Bigg(-\frac{2d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\Bigg)\Bigg)\Bigg)+\frac{1}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)^2}\times
$$

$$
\Bigg(\varphi(w)\Bigg(a\left(h_{j,k}\right)\Bigg(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\Bigg(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\Bigg)\Bigg)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}+
$$

$$
a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg(-\frac{d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)+
$$

$$\frac{1}{\left(\left(a\left(h_{j,k}\right)\right)^{2}\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)^{2}}\Bigg(v\varphi(w)\Bigg(\left(a\left(h_{j,k}\right)^{2}\right)\Bigg(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\Bigg(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^{2}}+$$

$$\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\Bigg)\Bigg)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}+\left(a\left(h_{j,k}\right)\right)^{2}\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\Bigg(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\times$$

$$\Bigg(-\frac{d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^{2}}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\Bigg)\Bigg)\Bigg)\Bigg)\Bigg(\Bigg(\Bigg(\frac{\Phi(w)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}+\frac{\varphi(w)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-$$

$$
\frac{\varphi(v)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}}\Bigg)\Bigg)\Bigg(\frac{\Phi(v)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}+\frac{\varphi(v)}{a\left(h_{j,k}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-
$$

$$
\frac{\varphi(w)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}\Bigg)+\frac{v\varphi(w)}{\left(a\left(h_{j,k}\right)^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}-
$$

$$
\frac{w\varphi(v)}{\left(a\left(h_{j,k}\right)^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}\Bigg)+\Bigg(\frac{\Phi(w)\left(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}+
$$

$$
\frac{\Phi(v)\Bigg(-\dfrac{d_k \ln\Big(1+d_k\beta_\zeta\Big(\dfrac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\Big)\Big)}{\big(d_k\beta_\zeta\big)^2}+\dfrac{d_k\Big(\dfrac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\Big)}{d_k\beta_\zeta\Big(1+d_k\beta_\zeta\Big(\dfrac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\Big)\Big)}\Bigg)}{\Big(1+d_k\beta_\zeta\Big(\dfrac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\Big)\Big)^{\frac{1}{d_k\beta_\zeta}}}\Bigg)\Bigg(\Bigg(\Bigg(\Bigg(\Bigg(\frac{2\Phi(w)\Bigg(d_j\beta_\zeta\Bigg(\dfrac{\big(x_{ji}-d_j\beta_\mu\big)d_j}{\big(d_j\beta_\sigma\big)^2}\Bigg)\Bigg)}{\Big(1+d_j\beta_\zeta\Big(\dfrac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\Big)\Big)^{\frac{2}{d_j\beta_\zeta}+1}d_j\beta_\zeta}+\frac{2\big(\varphi(w)\big)\Bigg(a\big(h_{j,k}\big)\Bigg(\dfrac{\Big(1+d_j\beta_\zeta\Big(\dfrac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\Big)\Big)^{\frac{2}{d_j\beta_\zeta}}\Bigg(d_j\beta_\zeta\Bigg(\dfrac{\big(x_{ji}-d_j\beta_\mu\big)d_j}{\big(d_j\beta_\sigma\big)^2}\Bigg)\Bigg)}{d_j\beta_\zeta\Big(1+d_k\beta_\zeta\Big(\dfrac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\Big)\Big)}\Bigg)\Bigg)}{\Bigg(a\big(h_{j,k}\big)\Big(1+d_j\beta_\zeta\Big(\dfrac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\Big)\Big)^{\frac{2}{d_j\beta_\zeta}}\Bigg)^2}+
$$

$$
\frac{\varphi(v)\Bigg(-a\big(h_{j,k}\big)\Bigg(-\dfrac{d_j\ln\Big(1+d_j\beta_\zeta\Big(\dfrac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\Big)\Big)}{\big(d_j\beta_\zeta\big)^2}+\dfrac{d_j\Big(\dfrac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\Big)}{d_j\beta_\zeta\Big(1+d_j\beta_\zeta\Big(\dfrac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\Big)\Big)}\Bigg)\Big(1+d_k\beta_\zeta\Big(\dfrac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\Big)\Big)^{\frac{1}{d_k\beta_\zeta}}-a\big(h_{j,k}\big)\Big(1+d_j\beta_\zeta\Big(\dfrac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\Big)\Big)^{\frac{1}{d_j\beta_\zeta}}\Bigg(\dfrac{\Big(1+d_k\beta_\zeta\Big(\dfrac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\Big)\Big)^{\frac{1}{d_k\beta_\zeta}}\Bigg(d_k\beta_\zeta\Bigg(\dfrac{\big(x_{ki}-d_k\beta_\mu\big)d_k}{\big(d_k\beta_\sigma\big)^2}\Bigg)\Bigg)}{d_k\beta_\zeta\Big(1+d_k\beta_\zeta\Big(\dfrac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\Big)\Big)}\Bigg)\Bigg)}{\Bigg(a\big(h_{j,k}\big)\Big(1+d_j\beta_\zeta\Big(\dfrac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\Big)\Big)^{\frac{1}{d_j\beta_\zeta}}\Big(1+d_k\beta_\zeta\Big(\dfrac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\Big)\Big)^{\frac{1}{d_k\beta_\zeta}}\Bigg)^2}\Bigg)\times
$$

$$\left(\frac{\Phi(v)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}+\frac{\varphi(v)}{a\left(h_{j,k}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(w)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}\right)+$$

$$\left(\frac{\Phi(w)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}+\frac{\varphi(w)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(v)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}}\right)\times$$

$$\frac{2\left(\Phi(v)\right)\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}+\frac{2\left(\varphi(w)\right)\left(a\left(h_{j,k}\right)\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}+$$

$$\left.\frac{\varphi(w)\left(-a\left(h_{j,k}\right)\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_k}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}-a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)^2}\right)-$$

$$\frac{1}{\left(\left(a\left(h_{j,k}\right)^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}\left(v\varphi(w)\left(-2\left(\left(a\left(h_{j,k}\right)\right)^2\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)d_j}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\times\right.\right.\right.\right.$$

$$\left.\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)-\left(a\left(h_{j,k}\right)\right)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left.\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\right)\right)+$$

$$\frac{1}{\left(\left(a\left(h_{j,k}\right)^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}\left(w\varphi(v)\left(\left(a\left(h_{j,k}\right)^2\right)\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\times\right.\right.$$

$$\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}-2\left(\left(a\left(h_{j,k}\right)\right)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left.\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\right)\right)\right)\right)\Bigg/$$

$$
\left(\left(\left(\frac{\Phi(w)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}+\frac{\varphi(w)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(v)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}}\right)\times\right.\right.
$$

$$
\left(\frac{\Phi(v)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}+\frac{\varphi(v)}{a\left(h_{j,k}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(w)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}\right)+
$$

$$
\left.\left.\frac{v\varphi(w)}{\left(a\left(h_{j,k}\right)^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}-\frac{w\varphi(v)}{\left(a\left(h_{j,k}\right)^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}\right)\right)+
$$

$$\left(-\frac{\Phi(w)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}-\frac{\Phi(v)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}\right)\left(\left(\left(\left(\frac{2(\Phi(w))\left(d_j\beta_\zeta\left(\frac{(x_{ji}-d_j\beta_\mu)d_j}{(d_j\beta_\sigma)^2}\right)\right)\left(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{(d_j\beta_\zeta)^2}+\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}(d_j\beta_\zeta)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}-\right.\right.\right.\right.$$

$$\frac{2(\Phi(w))\left(d_j\beta_\zeta\left(\frac{(x_{ji}-d_j\beta_\mu)d_j}{(d_j\beta_\sigma)^2}\right)\right)d_j}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}(d_j\beta_\zeta)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}+\frac{2(\Phi(w))\left(d_j\beta_\zeta\left(\frac{(x_{ji}-d_j\beta_\mu)d_j}{(d_j\beta_\sigma)^2}\right)\right)\left(d_j\left(\frac{(x_{ji}-d_j\beta_\mu)}{(d_j\beta_\sigma)^2}\right)\right)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}(d_j\beta_\zeta)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^2}-$$

$$
\frac{4(\varphi(w))\left(a\left(h_{j,k}\right)\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\left(a\left(h_{j,k}\right)\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\right)}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^3}+
$$

$$
\frac{1}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}\left(2(\varphi(w))\left(a\left(h_{j,k}\right)\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}-\right.\right.\right.
$$

$$
\left.\left.\left.\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)d_j}{\left(d_j\beta_\zeta\right)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}+\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}-\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\right)-
$$

$$\frac{1}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\right)^3}\Bigg(2\left(\varphi(v)\right)\Bigg(-a\left(h_{j,k}\right)\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\times$$

$$\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}-a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\Bigg)\Bigg)\Bigg(a\left(h_{j,k}\right)\Bigg(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\times$$

$$\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\Bigg)\Bigg)\Bigg(a\left(h_{j,k}\right)\Bigg(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+$$

$$\frac{\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\Bigg)\Bigg)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}+a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\times$$

$$\Bigg(-\frac{d_k\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\Bigg)\Bigg)\Bigg)\Bigg)+\frac{1}{\left(\left(a\left(h_{j,k}\right)^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\right)^2}\times$$

$$\Bigg(\varphi(v)\Bigg(-a\left(h_{j,k}\right)\Bigg(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}-$$

$$\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)d_j}{\left(d_j\beta_\zeta\right)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}+\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}-$$

$$\left.\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)\left(d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}-a\left(h_{j,k}\right)\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\times$$

$$\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(-\frac{d_k\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)-a\left(h_{j,k}\right)\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\times\right.$$

$$\Bigg(-\frac{d_j \ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\Bigg)\Bigg)\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)-$$

$$a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(-\frac{d_k \ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}-$$

$$\left.\left.\left.\left.\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)d_k}{\left(d_k\beta_\zeta\right)^2\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}+\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}-\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}\right)\right)\right)\right)\times$$

$$\left(\frac{\Phi(v)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}+\frac{\varphi(v)}{a\left(h_{j,k}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(w)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}\right)+$$

$$\left(\frac{2\left(\Phi(w)\right)\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)} + \frac{2\left(\varphi(w)\right)\left(a\left(h_{j,k}\right)\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2} \right)\Bigg)\Bigg) +$$

$$
\left.\frac{\varphi(v)\left(-a\left(h_{j,k}\right)\left(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}-a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\right)^2}\right)\times
$$

$$
\left(--\frac{\Phi(v)\left(-\frac{2d_k\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+\frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}-\frac{\varphi(v)\left(a\left(h_{j,k}\right)\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)\left(-\frac{2d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)}{\left(a\left(h_{j,k}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}+\right.
$$

$$
\frac{1}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)^2}\Bigg(\varphi(w)\Bigg(a\left(h_{j,k}\right)\Bigg(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+
$$

$$
\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\Bigg)\Bigg)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}+a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg(-\frac{d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+
$$

$$
\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)+\Bigg(-\frac{\Phi(w)\left(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-
$$

$$
\frac{\varphi(w)\left(a\left(h_{j,k}\right)\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\right)}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}+\frac{1}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\right)^2}\times
$$

$$
\frac{\varphi(v)\left(a\left(h_{j,k}\right)\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right.\right.}{\left(a\left(h_{j,k}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\right)^2}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}+
$$

$$
\left.a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}+\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(-\frac{d_k\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\right)\right)\times
$$

$$\left(\frac{2\left(\Phi(v)\right)\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}+\frac{2\left(\varphi(v)\right)\left(a\left(h_{j,k}\right)\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)}{\left(a\left(h_{j,k}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\right)^2}+\right.$$

$$\frac{\varphi(w)\left(a\left(h_{j,k}\right)\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}-a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\right)^2}+$$

$$\left(\frac{\Phi(w)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}+\frac{\varphi(w)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(v)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}}\right)\times$$

$$
\frac{2\left(\Phi(v)\right)\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)\left(-\frac{2d_k\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\sigma\right)^2}+\frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\sigma\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}-\frac{2\left(\Phi(v)\right)\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)d_k}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\right)^2\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}+
$$

$$
\frac{2\left(\Phi(v)\right)\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}-\frac{2\left(\Phi(v)\right)\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}-
$$

$$
\frac{4\left(\varphi(v)\right)\left(a\left(h_{j,k}\right)\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\left(a\left(h_{j,k}\right)\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(-\frac{2d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\right)}{\left(a\left(h_{j,k}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^3}+
$$

$$\frac{1}{\left(a\left(h_{j,k}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}\Bigg(2(\varphi(v))\Bigg(a\left(h_{j,k}\right)\Bigg(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\left(-\frac{2d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}-$$

$$\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)d_k}{\left(d_j\beta_\zeta\right)^2\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}+\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}-$$

$$\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}\Bigg)\Bigg)\Bigg)-$$

$$\frac{1}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)^3}\Bigg(2(\varphi(w))\Bigg(-a\left(h_{j,k}\right)\Bigg(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\times$$

$$\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}-a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\left(a\left(h_{j,k}\right)\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\times\right.\right.$$

$$\left.\left(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}+a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\times$$

$$\left.\left.\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\right)\right)+\frac{1}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)^2}\times$$

$$
\left(\varphi(w)\left(-a\left(h_{j,k}\right)\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}-\right.\right.\right.
$$

$$
\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)d_j}{\left(d_j\beta_\zeta\right)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}+\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}-
$$

$$
\left.\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}-a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\times
$$

$$
\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\Bigg)-
$$

$$
a\left(h_{j,k}\right)\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}-\right.
$$

$$
a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}-\right.
$$

$$\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)d_j}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}+\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}-\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)\left(d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)\times$$

$$\frac{1}{\left(\left(a\left(h_{j,k}\right)^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)^3}\Bigg(2\left(v\varphi(w)\right)\Bigg(-2\Bigg(\left(a\left(h_{j,k}\right)^2\right)\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\times$$

$$\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg)-\left(a\left(h_{j,k}\right)^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\Bigg)\times$$

$$\left(a\left(h_{j,k}\right)\right)^2\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}+$$

$$\left.\left.\left(a\left(h_{j,k}\right)\right)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\right)\right)\times$$

$$\frac{1}{\left(\left(a\left(h_{j,k}\right)^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)^2}\left(v\varphi(w)\right)\left(\left(-2\left(a\left(h_{j,k}\right)^2\right)\left(\frac{\left(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}-\right.\right.\right.$$

$$\left.\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{\left(d_j\beta_\zeta\right)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}+\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}-\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^2}\right)\times$$

$$\left.\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)-2\left(\left(a\left(h_{j,k}\right)\right)^2\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)-$$

$$\left.\left.\left.\left(\frac{d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\right)\right)\left(a\left(h_{j,k}\right)\right)^2\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\right.\right.$$

$$\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\Bigg)\Bigg)\Bigg)\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)-\left(a\left(h_{j,k}\right)\right)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\times$$

$$\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)-$$

$$\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)d_j}{\left(d_j\beta_\zeta\right)^2\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}+\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}-\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}\Bigg)\Bigg)\Bigg)-$$

$$\frac{1}{\left(\left(a\left(h_{j,k}\right)\right)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^3}\left(2\left(w\varphi(v)\right)\left(-\left(a\left(h_{j,k}\right)\right)^2\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right.\right.\times$$

$$\left.\left.\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}-2\left(\left(a\left(h_{j,k}\right)\right)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\right)\right)\times$$

$$\left(\left(a\left(h_{j,k}\right)\right)^2\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}+\right.$$

$$\left(a\left(h_{j,k}\right)\right)^{2}\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^{2}}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\Bigg)\Bigg)+$$

$$\frac{1}{\left(a\left(h_{j,k}\right)^{2}\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^{2}}\Bigg(w\varphi(v)\Bigg(-a\left(h_{j,k}\right)^{2}\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}+1}\left(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^{2}}+\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}-$$

$$\left(a\left(h_{j,k}\right)\right)^{2}\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(-\frac{2\left(d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)\right)}{\left(d_j\beta_\zeta\right)^{2}}+\frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\Bigg)-$$

$$
\left.\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)}\right)\right)d_j}{\left(d_j\beta_\zeta\right)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}+\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}-\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)}\right)\right)\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^2}\right)\times
$$

$$
2\left(\left(a\left(h_{j,k}\right)\right)^2\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)-
$$

$$
2\left(\left(a\left(h_{j,k}\right)\right)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(-\frac{2d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)-
$$

$$\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)d_j}{\left(d_j\beta_\zeta\right)^2\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}+\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}-\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)/$$

$$\Bigg(\Bigg(\Bigg(\frac{\Phi(w)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}+\frac{\varphi(w)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(v)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}}\Bigg)\times$$

$$\Bigg(\frac{\Phi(v)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}+\frac{\varphi(v)}{a\left(h_{j,k}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(w)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}\Bigg)+$$

$$
\left.\frac{v\varphi(w)}{\left(a\left(h_{j,k}\right)^{2}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}-\frac{w\varphi(v)}{\left(a\left(h_{j,k}\right)^{2}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}\right)-
$$

$$
\left(\left(\left(\left(\frac{2(\Phi(w))\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}+\frac{2(\varphi(w))\left(a\left(h_{j,k}\right)\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}+\right.\right.\right.\right.
$$

$$
\left.\frac{\varphi(v)\left(-a\left(h_{j,k}\right)\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}-a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\right)^2}\right)\times
$$

$$
\left(\frac{\Phi(v)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}+\frac{\varphi(v)}{a\left(h_{j,k}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(w)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}\right)+
$$

$$
\left(\frac{\Phi(w)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}+\frac{\varphi(w)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(v)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}}\right)\times
$$

$$
\left(\frac{2\left(\Phi(v)\right)\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\left(d_j\beta_\zeta\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}+\frac{2\left(\varphi(v)\right)\left(a\left(h_{j,k}\right)\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}+\right.
$$

$$\left.\frac{\varphi(w)\left(a\left(h_{j,k}\right)\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}-a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)^2}\right)-$$

$$\frac{1}{\left(\left(a\left(h_{j,k}\right)^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)^2}$$

$$\left(v\varphi(w)\left(-2\left(\left(a\left(h_{j,k}\right)^2\right)\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right.\right.\right.$$

$$\times$$

$$\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}-\left(a\left(h_{j,k}\right)^2\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\Bigg)\Bigg)\Bigg)+$$

$$\frac{1}{\left(\left(a\left(h_{j,k}\right)^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}\Bigg(w\varphi(v)\left(-\left(a\left(h_{j,k}\right)^2\right)\left(\frac{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(d_j\beta_\zeta\left(\frac{\left(x_{ji}-d_j\beta_\mu\right)d_j}{\left(d_j\beta_\sigma\right)}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\times$$

$$\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}-2\left(\left(a\left(h_{j,k}\right)\right)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\frac{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(d_k\beta_\zeta\left(\frac{\left(x_{ki}-d_k\beta_\mu\right)d_k}{\left(d_k\beta_\sigma\right)^2}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\Bigg)\Bigg)\Bigg)\times$$

$$\frac{\Phi(w)\left(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(w)\left(a\left(h_{j,k}\right)\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\right)}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}+$$

$$\frac{1}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\right)^2}\left(\varphi(v)\left(a\left(h_{j,k}\right)\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\right.\right.\right.\right.$$

$$\left.\left.\left.\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}+a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\times\right.$$

$$-\frac{d_k \ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg(\frac{\Phi(v)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}+\frac{\varphi(v)}{a\left(h_{j,k}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-$$

$$\frac{\varphi(w)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}\Bigg)+\Bigg(\frac{\Phi(w)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}+\frac{\varphi(w)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-$$

$$\frac{\varphi(v)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}}\Bigg)\Bigg(-\frac{\Phi(v)\left(-\frac{2d_k\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+\frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}-$$

$$\frac{\varphi(v)\left(a\left(h_{j,k}\right)\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\left(-\frac{2d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\right)}{\left(a\left(h_{j,k}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}+\frac{1}{\left(\left(a\left(h_{j,k}\right)^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}\times$$

$$\left(\varphi(w)\left(\left(a\left(h_{j,k}\right)^2\right)\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}+\right.\right.$$

$$\left.\left.\left(a\left(h_{j,k}\right)^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(-\frac{2d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\right)\right)+$$

$$\frac{1}{\left(\left(a\left(h_{j,k}\right)^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}\Bigg(w\varphi(v)\Bigg(\left(a\left(h_{j,k}\right)^2\right)\Bigg(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\times$$

$$\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}-\left(a\left(h_{j,k}\right)\right)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(-\frac{2\left(d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\Bigg)\Bigg)\Bigg)\Bigg)/$$

$$\left(\left(a\left(h_{j,k}\right)^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2/\Bigg(\Bigg(\Bigg(\frac{\Phi(w)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}+\frac{\varphi(w)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-$$

$$\frac{\varphi(v)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}}\Bigg)\Bigg(\frac{\Phi(v)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}+\frac{\varphi(v)}{a\left(h_{j,k}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-$$

$$\frac{\varphi(w)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}}\Bigg)+\frac{v\varphi(w)}{\left(a\left(h_{j,k}\right)^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}-$$

$$\frac{w\varphi(v)}{\left(a\left(h_{j,k}\right)^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}\Bigg)^2\Bigg)$$

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$$
\begin{aligned}
\frac{\partial^2 \ell(\boldsymbol{\beta})}{\partial\beta_\zeta \partial\beta_\zeta} = \sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m} \Bigg(& -\frac{\Phi(w)\left(-\dfrac{d_j \ln\left(1+d_j\beta_\zeta\left(\dfrac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\dfrac{d_j\left(\dfrac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\dfrac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)^2}{\left(1+d_j\beta_\zeta\left(\dfrac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}} \\
& +\frac{\Phi(w)\left(\dfrac{2d_j^{\,2}\ln\left(1+d_j\beta_\zeta\left(\dfrac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^3}-\dfrac{2d_j\left(d_j\left(\dfrac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}-\dfrac{\left(d_j\left(\dfrac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^2}{d_j\beta_\zeta}\right)}{\left(1+d_j\beta_\zeta\left(\dfrac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}} - \\
& \frac{\Phi(v)\left(-\dfrac{d_k \ln\left(1+d_k\beta_\zeta\left(\dfrac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+\dfrac{d_k\left(\dfrac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\dfrac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)^2}{\left(1+d_k\beta_\zeta\left(\dfrac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}} \\
& +\frac{\Phi(v)\left(\dfrac{2d_k^{\,2}\ln\left(1+d_k\beta_\zeta\left(\dfrac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^3}-\dfrac{2d_k\left(d_k\left(\dfrac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}-\dfrac{\left(d_j\left(\dfrac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\dfrac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)}{\left(1+d_k\beta_\zeta\left(\dfrac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}}\Bigg)\times
\end{aligned}
$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m}\ln\left(\left(\left(\left(\frac{\Phi(w)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}+\frac{\varphi(w)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(v)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}}\right)\times\right.\right.\right.$$

$$\left(\frac{\Phi(v)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}+\frac{\varphi(v)}{a\left(h_{j,k}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(w)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}\right)+$$

$$\left.\frac{v\varphi(w)}{\left(a\left(h_{j,k}\right)^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}-\frac{w\varphi(v)}{\left(a\left(h_{j,k}\right)^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}\right)+$$

$$
2\Bigg(\Bigg(\Phi(w)\left(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\Bigg/\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}+\frac{1}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}}\times
$$

$$
\Phi(v)\left(-\frac{d_k\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg(\Bigg(\Bigg(\Bigg(-\frac{\Phi(w)\left(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-
$$

$$
\frac{\varphi(w)\left(a\left(h_{j,k}\right)\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\right)}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}+
$$

$$\frac{1}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\right)^2}\Biggl(\varphi(v)\Biggl(a\left(h_{j,k}\right)\Biggl(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Biggl(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+$$

$$\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\Biggr)\Biggr)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}+a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Biggl(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\Biggl(-\frac{d_k\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+$$

$$\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\Biggr)\Biggr)\Biggr)\Biggr)\left(\frac{\Phi(v)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}+\frac{\varphi(v)}{a\left(h_{j,k}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(w)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}\right)+$$

$$\left(\frac{\Phi(w)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}} + \frac{\varphi(w)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}} - \frac{\varphi(v)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}} \right)\times$$

$$\left(-\frac{\Phi(v)\left(-\frac{2d_k\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2} + \frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}} - \frac{\varphi(v)\left(a\left(h_{j,k}\right)\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\left(-\frac{2d_k\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2} + \frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\right)}{\left(a\left(h_{j,k}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\right)^2} + \right.$$

$$\frac{1}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)^2}\left(\varphi(w)\left(a\left(h_{j,k}\right)\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2} + \right.\right.\right.\right.$$

$$\left.\left.\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}+a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\times\right.\right.$$

$$\left.\left.\left.\left.\left(-\frac{d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\right)\right)\right)-\frac{1}{\left(a\left(h_{j,k}\right)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}\times$$

$$\left(v\varphi(w)\right)\left(\left(a\left(h_{j,k}\right)^2\right)\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}+$$

$$\left(a\left(h_{j,k}\right)^{2}\right)\left(1+d_{j}\beta_{\zeta}\left(\frac{x_{ji}-d_{j}\beta_{\mu}}{d_{j}\beta_{\sigma}}\right)\right)^{\frac{2}{d_{j}\beta_{\zeta}}}\Bigg(\left(1+d_{k}\beta_{\zeta}\left(\frac{x_{ki}-d_{k}\beta_{\mu}}{d_{k}\beta_{\sigma}}\right)\right)^{\frac{1}{d_{j}\beta_{\zeta}}}\Bigg(\Bigg(-\frac{d_{j}\ln\left(1+d_{k}\beta_{\zeta}\left(\frac{x_{ki}-d_{k}\beta_{\mu}}{d_{k}\beta_{\sigma}}\right)\right)}{\left(d_{j}\beta_{\zeta}\right)^{2}}+\frac{d_{k}\left(\frac{x_{ki}-d_{k}\beta_{\mu}}{d_{k}\beta_{\sigma}}\right)}{d_{j}\beta_{\zeta}\left(1+d_{k}\beta_{\zeta}\left(\frac{x_{ki}-d_{k}\beta_{\mu}}{d_{k}\beta_{\sigma}}\right)\right)}\Bigg)\Bigg)\Bigg)\Bigg)+$$

$$\frac{1}{\left(\left(a\left(h_{j,k}\right)^{2}\right)\left(1+d_{j}\beta_{\zeta}\left(\frac{x_{ji}-d_{j}\beta_{\mu}}{d_{j}\beta_{\sigma}}\right)\right)^{\frac{1}{d_{j}\beta_{\zeta}}}\left(1+d_{k}\beta_{\zeta}\left(\frac{x_{ki}-d_{k}\beta_{\mu}}{d_{k}\beta_{\sigma}}\right)\right)^{\frac{2}{d_{j}\beta_{\zeta}}}\right)^{2}}\Bigg(w\varphi(v)\Bigg(\left(a\left(h_{j,k}\right)^{2}\right)\Bigg(\left(1+d_{j}\beta_{\zeta}\left(\frac{x_{ji}-d_{j}\beta_{\mu}}{d_{j}\beta_{\sigma}}\right)\right)^{\frac{1}{d_{j}\beta_{\zeta}}}\Bigg(-\frac{d_{j}\ln\left(1+d_{j}\beta_{\zeta}\left(\frac{x_{ji}-d_{j}\beta_{\mu}}{d_{j}\beta_{\sigma}}\right)\right)}{\left(d_{j}\beta_{\zeta}\right)^{2}}+$$

$$\frac{d_{j}\left(\frac{x_{ji}-d_{j}\beta_{\mu}}{d_{j}\beta_{\sigma}}\right)}{d_{j}\beta_{\zeta}\left(1+d_{j}\beta_{\zeta}\left(\frac{x_{ji}-d_{j}\beta_{\mu}}{d_{j}\beta_{\sigma}}\right)\right)}\Bigg)\Bigg)\Bigg)\left(1+d_{k}\beta_{\zeta}\left(\frac{x_{ki}-d_{k}\beta_{\mu}}{d_{k}\beta_{\sigma}}\right)\right)^{\frac{2}{d_{j}\beta_{\zeta}}}+\left(a\left(h_{j,k}\right)\right)^{2}\left(1+d_{j}\beta_{\zeta}\left(\frac{x_{ji}-d_{j}\beta_{\mu}}{d_{j}\beta_{\sigma}}\right)\right)^{\frac{1}{d_{j}\beta_{\zeta}}}\Bigg(\left(1+d_{k}\beta_{\zeta}\left(\frac{x_{ki}-d_{k}\beta_{\mu}}{d_{k}\beta_{\sigma}}\right)\right)^{\frac{2}{d_{j}\beta_{\zeta}}}\times$$

$$\left(-2d_j \ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)/\left(d_j\beta_\zeta\right)^2+d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)/d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)\right)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)/$$

$$\left(\left(\left(\frac{\Phi(w)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}+\frac{\varphi(w)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(v)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}}\right)\times\right.\right.$$

$$\left(\frac{\Phi(v)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}+\frac{\varphi(v)}{a\left(h_{j,k}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(w)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}\right)+$$

$$\frac{v\varphi(w)}{\left(a\left(h_{j,k}\right)\right)^{2}\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}-\frac{w\varphi(v)}{\left(a\left(h_{j,k}\right)\right)^{2}\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}\Bigg)+\Bigg(-\frac{\Phi(w)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}-\frac{\Phi(v)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}}\Bigg)\times$$

$$\Bigg(\Bigg(\Bigg(\Bigg(\frac{\Phi(w)\left(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^{2}}+\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)^{2}}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\Phi(w)\left(\frac{4d_j^{2}\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^{3}}-\frac{4d_j\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^{2}}-\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{2}}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{2}}\right)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}+$$

$$
\frac{2\left(\varphi(w)\right)\left(a\left(h_{j,k}\right)\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\right)^2}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^3}-\frac{1}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}\times
$$

$$
\left(\varphi(w)\left(a\left(h_{j,k}\right)\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)^2+\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\times\right.\right.\right.
$$

$$
\left.\left.\left.\left(\frac{4d_j^{\,2}\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^3}-\frac{4d_j\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}-\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^2}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^2}\right)\right)\right)\right)-\frac{1}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\right)^3}\times
$$

$$\Bigg(2\left(\varphi(v)\right)\Bigg(a\left(h_{j,k}\right)\Bigg(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\Bigg)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}+$$

$$a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\Bigg(\Bigg(-\frac{d_k\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\Bigg)\Bigg)\Bigg)^2\Bigg)+$$

$$\frac{1}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\right)^2}\Bigg(\varphi(v)\Bigg(a\left(h_{j,k}\right)\Bigg(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+$$

$$\left.\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)^2+\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\frac{2d_j^{\,2}\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^3}-\frac{2d_j\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}-\frac{\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^2}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^2}\right)\right)\times$$

$$\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}+2\left(a\left(h_{j,k}\right)\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\right)\times$$

$$\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\left(-\frac{d_k\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\Bigg)\Bigg)+a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\times$$

$$\Bigg(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}} \left(-\frac{d_k \ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2} + \frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)} \right)^2 + \left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}} \times$$

$$\left(\frac{2d_k{}^2 \ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^3} - \frac{2d_k\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2} - \frac{\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2} \right)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg(\Bigg(\Bigg(\frac{\Phi(v)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}} +$$

$$\frac{\varphi(v)}{a\left(h_{j,k}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}} - \frac{\varphi(w)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}} \Bigg) + 2\Bigg(\Bigg(\frac{\Phi(w)\left(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2} + \frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}} -$$

$$\frac{\varphi(w)\Bigg(a\left(h_{j,k}\right)\Bigg(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\Bigg(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\Bigg)\Bigg)\Bigg)}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}+\frac{1}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\right)^2}\times$$

$$\Bigg(\varphi(v)\Bigg(a\left(h_{j,k}\right)\Bigg(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\Bigg)\Bigg)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}+$$

$$a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\Bigg(-\frac{d_k\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\Bigg)\Bigg)\Bigg)\Bigg)\times$$

$$\left(\frac{\Phi(v)\left(-\frac{2d_k\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+\frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}-\frac{\varphi(v)\left(a\left(h_{j,k}\right)\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\left(-\frac{2d_k\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+\frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\right)}{\left(a\left(h_{j,k}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\right)^2}+\right.$$

$$\frac{1}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)^2}\left(\varphi(w)\left(a\left(h_{j,k}\right)\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\right.\right.\right.\right.$$

$$\left.\left.\left.\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}+a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\right.\right.$$

$$\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)+\left(\frac{\Phi(w)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}+\frac{\varphi(w)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(v)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}}\right)\times$$

$$\frac{\Phi(v)\left(-\frac{2d_k\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+\frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)^2}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}-\frac{\Phi(v)\left(\frac{4{d_k}^2\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^3}-\frac{4d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{\left(d_k\beta_\zeta\right)^2}-\frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}+$$

$$\frac{2\left(\varphi(v)\right)\left(a\left(h_{j,k}\right)\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\left(\left(-\frac{2d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\right)\right)^2}{\left(a\left(h_{j,k}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^3}-\frac{1}{\left(a\left(h_{j,k}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}\times$$

$$\Bigg(\varphi(v)\Bigg(a\left(h_{j,k}\right)\Bigg(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)^2+\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}\times$$

$$\left(\frac{4d_j^{\,2}\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^3}-\frac{4d_j\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}-\frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}\right)\Bigg)\Bigg)\Bigg)-\frac{1}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)^3}\times$$

$$\Bigg(2\left(\varphi(w)\right)\Bigg(a\left(h_{j,k}\right)\Bigg(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\Bigg)\Bigg)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}+$$

$$a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)^2\Bigg)+$$

$$\frac{1}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)^2}\Bigg(\varphi(w)\Bigg(a\left(h_{j,k}\right)\Bigg(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+$$

$$\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\Bigg)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\frac{2d_j^2\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^3}-\frac{2d_j\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}-\frac{\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^2}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^2}\right)\Bigg)\Bigg)\times$$

$$\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}+2\left(a\left(h_{j,k}\right)\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\times\right.$$

$$\left.\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\right)+a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\times$$

$$\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)^2+\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\times\right.$$

$$
\Bigg(\frac{2d_j^{\,2}\ln\Big(1+d_k\beta_\zeta\Big(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\Big)\Big)}{(d_j\beta_\zeta)^3}-\frac{2d_j\Big(d_k\Big(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\Big)\Big)}{(d_j\beta_\zeta)^2\Big(1+d_k\beta_\zeta\Big(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\Big)\Big)}-\frac{\Big(d_k\Big(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\Big)\Big)^2}{d_j\beta_\zeta\Big(1+d_k\beta_\zeta\Big(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\Big)\Big)^2}\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)+\frac{1}{\Bigg(\Big(a\big(h_{j,k}\big)^2\Big)\Big(1+d_j\beta_\zeta\Big(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\Big)\Big)^{\frac{2}{d_j\beta_\zeta}}\Big(1+d_k\beta_\zeta\Big(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\Big)\Big)^{\frac{1}{d_j\beta_\zeta}}\Bigg)^3}\times
$$

$$
\Bigg(2\big(v\varphi(w)\big)\Bigg(\Big(a\big(h_{j,k}\big)^2\Big)\Bigg(\Big(1+d_j\beta_\zeta\Big(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\Big)\Big)^{\frac{2}{d_j\beta_\zeta}}\Bigg(-\frac{2d_j\ln\Big(1+d_j\beta_\zeta\Big(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\Big)\Big)}{(d_j\beta_\zeta)^2}+\frac{2\Big(d_j\Big(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\Big)\Big)}{d_j\beta_\zeta\Big(1+d_j\beta_\zeta\Big(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\Big)\Big)}\Bigg)\Bigg)\Bigg)\Big(1+d_k\beta_\zeta\Big(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\Big)\Big)^{\frac{1}{d_j\beta_\zeta}}+
$$

$$
\Big(a\big(h_{j,k}\big)^2\Big)\Big(1+d_j\beta_\zeta\Big(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\Big)\Big)^{\frac{2}{d_j\beta_\zeta}}\Bigg(\Big(1+d_k\beta_\zeta\Big(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\Big)\Big)^{\frac{1}{d_j\beta_\zeta}}\Bigg(-\frac{d_j\ln\Big(1+d_k\beta_\zeta\Big(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\Big)\Big)}{(d_j\beta_\zeta)^2}+\frac{d_k\Big(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\Big)}{d_j\beta_\zeta\Big(1+d_k\beta_\zeta\Big(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\Big)\Big)}\Bigg)\Bigg)\Bigg)\Bigg)^2\Bigg)-
$$

$$\frac{1}{\left(\left(a\left(h_{j,k}\right)^{2}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)^{2}}\Bigg(v\varphi(w)\Bigg(\left(a\left(h_{j,k}\right)^{2}\right)\Bigg(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\Bigg(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^{2}}+$$

$$\frac{2d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\Bigg)^{2}+\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\Bigg(\frac{4d_j^{\,2}\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^{3}}-\frac{4d_j\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^{2}\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}-$$

$$\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{2}}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{2}}\Bigg)\Bigg)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}+2\Bigg(\left(a\left(h_{j,k}\right)\right)^{2}\Bigg(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\Bigg(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^{2}}+$$

$$\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\Bigg)\Bigg)\Bigg(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\Bigg)\Bigg)\Bigg)+$$

$$\left(a\left(h_{j,k}\right)\right)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\Bigg(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)^2+$$

$$\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\frac{2d_j^{\,2}\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^3}-\frac{2d_j\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}-\frac{\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}\right)\Bigg)\Bigg)\Bigg)\Bigg)-$$

$$\frac{1}{\left(\left(a\left(h_{j,k}\right)^{2}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^{3}}\Bigg(2\left(w\varphi(v)\right)\Bigg(\left(a\left(h_{j,k}\right)^{2}\right)\Bigg(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^{2}}+$$

$$\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\Bigg)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}+\left(a\left(h_{j,k}\right)\right)^{2}\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\times$$

$$\Bigg(-\frac{2d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^{2}}+\frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\Bigg)\Bigg)\Bigg)\Bigg)^{2}\Bigg)+\frac{1}{\left(\left(a\left(h_{j,k}\right)^{2}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^{2}}\times$$

$$
\Bigg(w\varphi(v) \Bigg(\left(a\left(h_{j,k}\right)\right)^2 \Bigg(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}} \left(-\frac{d_j \ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2} + \frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)} \right)^2 + \left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}} \times
$$

$$
\left(\frac{2{d_j}^2 \ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^3} - \frac{2d_j\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)} - \frac{\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^2}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^2} \right) \Bigg) \left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}} +
$$

$$
2\Bigg(\left(a\left(h_{j,k}\right)\right)^2 \Bigg(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}} \left(-\frac{d_j \ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2} + \frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)} \right) \Bigg) \Bigg) \Bigg(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}} \times
$$

$$\left(-\frac{2d_j \ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\Bigg)\Bigg)\Bigg)+\left(a\left(h_{j,k}\right)\right)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\times\right.$$

$$\left(\frac{4d_j^{\,2}\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^3}-\frac{4d_j\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{\left(d_j\beta_\zeta\right)^2\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}-\frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^2}\right)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg/\left(\left(\frac{\Phi(w)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}+\frac{\varphi(w)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\right.\right.$$

$$\left.\frac{\varphi(v)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}}\right)\left(\frac{\Phi(v)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}+\frac{\varphi(v)}{a\left(h_{j,k}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\right.$$

$$\left.\frac{\varphi(w)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}\right)+\frac{v\varphi(w)}{\left(a\left(h_{j,k}\right)^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}-$$

$$\left.\frac{w\varphi(v)}{\left(a\left(h_{j,k}\right)^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}\right)-\left(\left(\left(\left(-\frac{\Phi(w)\left(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\right.\right.\right.\right.$$

$$\frac{\varphi(w)\left(a\left(h_{j,k}\right)\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\right)}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}+$$

$$\frac{1}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\right)^2}\Bigg(\varphi(v)\Bigg(a\left(h_{j,k}\right)\Bigg(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+$$

$$\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\Bigg)\Bigg)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}+a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\Bigg(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}\times$$

$$-\frac{d_k\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg(\frac{\Phi(v)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}+\frac{\varphi(v)}{a\left(h_{j,k}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-$$

$$\left.\frac{\varphi(w)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}\right)+\left(\frac{\Phi(w)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}+\frac{\varphi(w)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\right.$$

$$\left.\frac{\varphi(v)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}}\right)\left(-\frac{\Phi(v)\left(-\frac{2d_k\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_k\beta_\zeta\right)^2}+\frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_k\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}-\right.$$

$$\left.\frac{\varphi(v)\left(a\left(h_{j,k}\right)\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)\left(-\frac{2d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)}{\left(a\left(h_{j,k}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}\right)\Bigg)+$$

$$\frac{1}{\left(a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)^2}\left(\varphi(w)\left(a\left(h_{j,k}\right)\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\right.\right.\right.\right.$$

$$\left.\left.\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}+a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\right.\right.$$

$$\left.\left.\left.\left.\left.\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\right)\right)\right)-\frac{1}{\left(\left(a\left(h_{j,k}\right)^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\right)^2}\left(v\varphi(w)\left(\left(a\left(h_{j,k}\right)^2\right)\times\right.\right.$$

$$\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(-\frac{2d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}+$$

$$\left.\left(a\left(h_{j,k}\right)\right)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\right)\right)+$$

$$\frac{1}{\left(\left(a\left(h_{j,k}\right)^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\right)^2}\left(w\varphi(v)\left(\left(a\left(h_{j,k}\right)^2\right)\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(-\frac{d_j\ln\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\right.\right.\right.\right.$$

$$\left.\left.\left.\frac{d_j\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)}{d_j\beta_\zeta\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)}\right)\right)\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}+\left(a\left(h_{j,k}\right)\right)^2\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\times\right.$$

$$\left.\left.\left.\left.\left.\left(-\frac{2d_j\ln\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{\left(d_j\beta_\zeta\right)^2}+\frac{2\left(d_k\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}{d_j\beta_\zeta\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)}\right)\right)\right)\right)\right)\Big/\left(\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}-1}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}-1}\right)^2\Bigg)^2\Big/$$

$$\left(\left(\left(\frac{\Phi(w)}{\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}+\frac{\varphi(w)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(v)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_k\beta_\zeta}}}\right)\times\right.\right.\right.$$

$$\left(\frac{\Phi(v)}{\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_k\beta_\zeta}}}+\frac{\varphi(v)}{a\left(h_{j,k}\right)\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}-\frac{\varphi(w)}{a\left(h_{j,k}\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}\right)+$$

$$\left.\left.\frac{v\varphi(w)}{\left(a\left(h_{j,k}\right)^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}}-\frac{w\varphi(v)}{\left(a\left(h_{j,k}\right)^2\right)\left(1+d_j\beta_\zeta\left(\frac{x_{ji}-d_j\beta_\mu}{d_j\beta_\sigma}\right)\right)^{\frac{1}{d_j\beta_\zeta}}\left(1+d_k\beta_\zeta\left(\frac{x_{ki}-d_k\beta_\mu}{d_k\beta_\sigma}\right)\right)^{\frac{2}{d_j\beta_\zeta}}}\right)^2\right)$$

Lampiran 25 Tabel Aderson Darling

n	***α***							
	0,250	**0,150**	**0,100**	**0,050**	**0,025**	**0,010**	**0,005**	**0,001**
10	1,2419	1,6277	1,9518	2,5121	3,0990	3,9083	4,5175	5,9897
20	1,2500	1,6290	1,9385	2,5020	3,0731	3,8995	4,5117	5,9852
30	1,2457	1,6210	1,9313	2,5130	3,1111	3,9673	4,5309	5,8924
40	1,2450	1,6173	1,9362	2,5042	3,1047	3,9397	4,5889	6,1275
50	1,2425	1,6163	1,9277	2,4941	3,0933	3,9200	4,5211	5,9437
60	1,2464	1,6225	1,9367	2,5044	3,0776	3,9234	4,4858	6,0808
70	1,2515	1,6245	1,9304	2,4959	3,0889	3,8673	4,5326	5,9428
80	1,2384	1,6148	1,9235	2,4951	3,0778	3,8458	4,4808	5,9249
90	1,2461	1,6177	1,9326	2,5064	3,1020	3,9239	4,5856	6,0412
100	1,2399	1,6235	1,9235	2,4901	3,0655	3,8319	4,4068	5,8987
Mean	**1,2453**	**1,6211**	**1,9355**	**2,4986**	**3,0916**	**3,9033**	**4,5416**	**6,0255**

BIOGRAF I PENULIS

Penulis memiliki nama lengkap Siti Azizah, lahir di Pati pada tangal 23 Juni 1992. Penulis merupakan anak pertama dari pasangan Nur Arif Junaedi S.Ip., S.E., S.Pd. dan Wahyu Widayati S.Pd.. Penulis resmi menjadi istri dari Rizal Faiz Mohammad S.Pd.I. mulai tahun 2014 hingga sekarang. Penulis menempuh pendidikan formal sejak tahun 1998 di SDN 1 Pecangaan Wetan, dilanjutkan di SMPN 1 Pecangaan yang selesai tahun 2007, dan di SMAN 1 Pecangaan hingga tahun 2010. Penulis meraih gelar Sarjana Sains dari Jurusan Statistika Universitas Diponegoro pada tahun 2014 dan pada tahun yang sama diterima secara resmi sebagai mahasiswa Program Magister Jurusan Statistika Institut Teknologi Sepuluh Nopember Surabaya. Penulis dapat dihubungi melalui alamat email sitiazizah1223@gmail.com.